MATH MIDWAY 2GO

NUMBER LINE TIGHTROPE

Lesson Plan

GRADE 1

Brought to you by MO MATH

A Letter from the PSEG Foundation

My fascination with energy started at a young age.

The Arab oil embargo of the 1970's sent gasoline prices through the roof and made clear how closely tied our country's foreign policy is to oil interests. I began wondering whether we could produce energy in ways that didn't involve oil, and I wanted to be part of the quest to find the answer.

That passion led me to pursue years of study in the fields of physics and engineering. Graduate degrees in those areas allowed me to take on a variety of fascinating assignments in my career. I served as a research scientist at the Princeton Plasma Physics Lab, a Congressional Science Fellow in the office of U.S. Senator Bill Bradley, and a science, energy, and technology policy advisor to Governor Tom Kean before coming to PSEG where I work every day to create and deliver power responsibly.

This curriculum, developed by the Museum of Mathematics and funded by PSEG, is intended to help young people develop an interest in math and the technical fields–to spark curiosity, stimulate inquiry, and help students down a path of discovery that leads to fulfilling careers.

As issues such as climate change, energy independence, and national security demand increasingly comprehensive and technical solutions, the need for people with knowledge in science, technology, engineering, and math–areas known as the STEM subjects–will continue to grow.

At PSEG, we understand that our country's future depends on developing the insights, creativity, and dynamism of the next generation of innovators. This curriculum is one of many investments we've made in an effort to help young people discover their talents and develop a thirst for knowledge.

A math- and science-savvy workforce will lead the way to innovative technological discovery, a strengthened economy, and thriving new industries. And it is an important part of building a talent pipeline for the energy industry and our country as a whole.

Ralph Izzo
Chairman, CEO and President, PSEG

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Alfred P. Sloan Foundation in the creation of *Math Midway 2 Go*, and the support
of the PSEG Foundation in the creation of the accompanying curriculum.

Alfred P. Sloan Foundation

PSEG Foundation

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General Instructions for *Math Midway 2 Go*

Math Midway 2 Go (MM2GO) consists of six interactive mathematics exhibits that can travel to schools and other venues. Hands-on activities captivate and engage students, highlighting the wonder of mathematics. These exhibits were designed for use with individuals of all ages, and the mathematical topics they address range from topics in the elementary classroom to college-level mathematics. Students of all ages will benefit from open exploration of the exhibits. At the same time, the exhibits also tie into specific curricular topics for kindergarten through grade 12.

These lesson plans are provided by MoMath to support teachers like you. To help you and your students make the most of your time at *Math Midway 2 Go*, a focus exhibit has been selected for each grade from kindergarten though grade 12. The Grade 1 focus exhibit is the *Number Line Tightrope*.

MM2GO is designed to accommodate one class of up to 36 students at a time.

It is ideal to have only a small group of students at each exhibit while visiting *Math Midway 2 Go*. Break your class into six groups and have them rotate through the exhibits, with one group at each exhibit at a time. Before starting, make sure that students understand basic rules for interacting with the exhibits:

- Walk in the area surrounding the exhibits; don't run.
- Handle the exhibits gently.
- Do not hang or lean on the *Number Line Tightrope*.
- Handle *Ring of Fire* shapes gently.

Ideally, school support staff and/or parent volunteers will be available for the duration of the visit to *Math Midway 2 Go*. These adults can circulate throughout the exhibits, while the classroom teacher remains at the focus exhibit. At the five exhibits that are not the grade-level focus, students can explore and play.

Information about the *Number Line Tightrope*

About the exhibit:

The *Number Line Tightrope* features the numbers from -10 to 100, many of which are decorated with colorful hanging iconic shapes, arranged on a long horizontal beam. Each icon represents a different number family. As students explore, they link their observations with their prior knowledge of numbers and their properties. For example, students might try to figure out why certain numbers have a square hanging from them, while others do not. There are fourteen different number families to explore. Some are quite challenging, but even the youngest students can understand the square numbers, or learn why we call some numbers "triangular."

Why visit the *Number Line Tightrope*?

Counting is a major part of first grade mathematics, as students expand their knowledge of numbers above 100 and solidify their mental addition and subtraction skills. As students learn to count, they come to understand the role of the number line as a tool that supports counting skills. Exploring the *Number Line Tightrope* is an early exploration of how number lines work.

At the same time, students can use their observation skills to explore patterns along the number line. While the specific number families depicted relate to concepts that students will learn later, some of the number families, like triangular numbers, follow a pattern that will provide a fun yet achievable challenge for first graders.

The triangular numbers begin 1, 3, 6, 10, 15... What is the relationship between one triangular number and the next? How far away will the next triangular number be found? Students can follow the number line to check their predictions, gaining confidence in their math abilities.

Integrating MM2GO Into Your Unit Plans

Consider the following key questions, class topics, and elements of Common Core State Standards when considering how to link the *Number Line Tightrope* to the study of mathematics taking place in your classroom.

Key questions inspired by the *Number Line Tightrope*:

- What is a number line? Why would you write numbers on a line? What can you learn about numbers from a number line?
- What numbers are related to each other, and how?
- How can I describe number patterns using words, numbers, operations, movements, pictures, and/or art?
- What are some interesting number patterns that mathematicians study? How can I make an interesting number pattern?

This lesson plan will be useful with the following classes:

- Classes studying the order of numbers
- Classes learning the operations of addition and subtraction, and looking for a way to apply that learning
- Classes learning to group numbers by different properties and patterns

Relevant connections to the Common Core State Standards:

Learning Standards

1.OA: Represent and solve problems involving addition and subtraction.

Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

Number Line Tightrope Pre-Activity

Description

In this activity, students will explore number lines, focusing on their organization and how they can be used.

While this activity is designed for use before visiting the *Number Line Tightrope*, the activity can be enjoyed independently of a visit from the Museum of Mathematics' *Math Midway 2 Go*.

Materials

- Pencils
- Attached *Explore The Number Line* sheet, one copy for each student
- A large number line, attached to the floor of the classroom in a visible location
  To make a number line, tape a long strip (or several strips next to each other) of wide masking tape to the floor of your classroom. Write numbers evenly-spaced on the masking tape with a dark marker. Be sure that the numbers are spaced widely enough so that a student can clearly stand on any one particular number. Or, tape sheets of blank paper together, draw a number line on the paper, and tape to the floor of your classroom.

Key Terminology

- **Number line**
- **Add**
- **Greater than**
- **Less than**
- **Pattern**

Conducting the Activity

1. Seat students in pairs. Distribute one copy of *Explore The Number Line* to each student. Give students time to work on the sheet individually, and then to share their work with their partners.

2. If some students finish before others, have these students extend one of the activities on the worksheet, this time by skipping two numbers, and putting a triangle around the next number, skipping two numbers, and repeating until the number line ends. Ask students: what do you to do to the first number on the list to get the second number? Students will start to see the link between skipping two numbers and adding two if they are already learning addition.

Number Line Tightrope Pre-Activity (Continued)

3. After all students have finished the sheet and discussed their work with their partners, ask students to describe for their classmates the diagrams on the worksheet. Point out that the numbers are placed along a straight line and ask students: how are the numbers placed on the line? Use the word **number line** to describe the diagram. Ask students: why would we want to put numbers on a line in this way? What can we learn about numbers by writing them along a line? Pick a number. Where do numbers **greater than** that number go on the line? Where do numbers **less than** that number go on the line? Help students come to the conclusion that number lines are ordered left-to-right from smaller numbers to bigger numbers.

4. Next, have a class discussion about the answers on the worksheet. How are the circled numbers on the number line related? Share descriptions of what you do to the first number on the list to get the second number on the list. Encourage students to use language of counting and addition: you count up two or add two to the first number to get the next number. Ask students, which numbers would you mark if you counted up or added three to the first number, and so on? Which numbers would you mark if you counted up or added one to the first number, and so on?

5. Explain to students that when they listed the numbers that they circled, they were listing the numbers in a **pattern**. Work as a class to describe what makes a pattern.

6. How can you describe a pattern to another person? Tell students that now they're going to play a game called the Number Pattern Boogie, where they are going to show patterns with their actions!

7. Invite one student to the front of the room and have him or her stand in the space before the number one on the number line you have placed on the floor of the classroom. Have the student face the class with his or her back to the blackboard. Explain that you are going to write a secret number pattern there, and he or she cannot peek.

Number Line Tightrope Pre-Activity (Continued)

8. Write on the blackboard a circle-skip number pattern that shows where to start and how many numbers to skip before the next number in the pattern. An example might be to start at four and skip five numbers each time. Ask the class: can someone give this student instructions for how to jump along the number line and land on the numbers in this pattern, without listing the numbers? Have the student standing on the line try out the suggestions until he or she skips and lands on the correct numbers and make it to the end of the number line. Make sure that the student boogies along the line! This is the Number Pattern Boogie–skipping along the number line and landing on the correct numbers based on a pattern.

9. Now, students get to Number Pattern Boogie with a partner. In their pairs, students will pick a number pattern. It can be something like circle a number, skip a number, or it can be more complicated, as long as the pattern is consistent. Then, they are going to write instructions for the Number Pattern Boogie they invented. Their instructions should be as clear and precise as possible–because they are going to give their instructions to another pair, and those students are going to perform the Boogie for the class.

10. Give students time to write their sets of instructions. Make sure the pairs try their instructions before turning them in. As they work, walk around the room asking students questions about their instructions. Could their instructions be followed to make more than one number pattern? Do their instructions specify where to start? Encourage students to use language of addition when possible.

11. Have pairs trade instructions. If you have time, each pair can perform the Boogie for the entire class. If not, the pairs that traded can perform for each other. You will need additional number lines on the floor for this option.

12. After the performances, students should return to their seats with their partners. Have students share some of the number patterns they were given. Ask students, what was challenging about writing instructions? What was challenging about following the instructions? How can we make instructions better for next time?

Number Line Tightrope Pre-Activity (Continued)

Extension: Change it Up

Can you make different number lists using the same circle-skip pattern but with different starting numbers? Have students practice and see what happens.

Extension: Number Line Art

Give students a sheet with a number line on it with the numbers written in boxes. 50 is a good place to stop. Have students choose number patterns and then color in the boxes with the numbers that fit the pattern. What visual patterns do they get? Have students choose a new pattern, and color that in on the same sheet. Look for overlaps in the colors, and patterns in the overlaps. This can also be done using a 100s chart.

Number Line Tightrope Activity

Description

In this activity, students will explore the *Number Line Tightrope*, exploring the number pattern they see.

Materials

- Attached *Number Line Tightrope* recording sheet, one copy per student
- Pencils
- Optional: clipboards

Key Terminology

- **Number line**
- **Greater than**
- **Less than**
- **Pattern**

Conducting the Activity

1. Explain to students that as they look at the *Number Line Tightrope*, they should search for number patterns to share with their classmates.

2. Allow students to explore the *Number Line Tightrope* at their own pace. As they explore, circulate and talk to students. Ask them what they notice about the number patterns they see.

3. After some time for free exploration, gather students again. Ask them to share interesting things they noticed about the *Number Line Tightrope*. They will probably mention the dangling symbols. Ask them: why do you think those are on the *Number Line Tightrope*? What do you think the symbols mean?

4. Before students return to the *Number Line Tightrope*, ask each student to choose a symbol. Hand out the *Number Line Tightrope* recording sheet. Students will hunt for the symbol they selected and record where they find it.

5. Students can now go back to the *Number Line Tightrope* and hunt for their symbol. If time permits, students can choose a second symbol to hunt for as well.

Number Line Tightrope Activity (Continued)

6. After students complete their hunts, gather them together again. Share with each other the numbers and symbols they found. Collect their lists for use in the post-activity. Explain to students that back in the classroom, they will continue to explore the number patterns they found on the *Number Line Tightrope*.

Number Line Tightrope Post-Activity

Description

In this activity, students will use their observations from visiting the *Number Line Tightrope* to create more complicated Number Pattern Boogies.

While this activity is designed for use after visiting the *Number Line Tightrope*, the activity can be conducted with students who have not had the opportunity to experience the Museum of Mathematics' *Math Midway 2 Go*.

Materials

- *Number Line Tightrope* recording sheet, from the exhibit observation
- Attached *Number Line Tightrope Number Family Explanations and Examples*, one copy for the teacher
- Paper
- Pencils
- One number line sheet for each student
- A large number line, attached to the floor of the classroom in a visible location

  To make a number line, tape a long strip (or several strips next to each other) of wide masking tape to the floor of your classroom. Write numbers evenly-spaced on the masking tape with a dark marker. Be sure that the numbers are spaced widely enough so that a student can clearly stand on any one particular number. Or, tape sheets of blank paper together, draw a number line on the paper, and tape to the floor of your classroom.

Key Terminology

- **Number line**
- **Add**
- **Pattern**

Conducting the Activity

1. Return the *Number Line Tightrope* recording sheet to students. As a class, make a list of all the symbols students investigated and the numbers where those symbols were found.

2. Revisit the definition of a number pattern. Ask students, are these number patterns? What makes them different from the number patterns we looked at in the pre-activity? What makes them the same? Have a class discussion.

Number Line Tightrope Post-Activity (Continued)

Very few of the Number Line Tightrope patterns are based on simple counting and skipping. However, you can look at the amounts that are added to get from one number in a pattern to the next, particularly for triangular numbers and pizza numbers. See if students notice a pattern in the amounts that are added to get from one triangular number to the next. How about the pizza numbers?

3. Explain to students that they're going to make Number Boogie instructions for these number lists. Before students begin working, go over the suggestions they developed for making good instructions during the pre-activity. These will be especially important now, because these number patterns are much more complicated!

4. Each pair will pick one of the lists to use. Students should again write down their instructions. This time, because the number lists are complicated, the instructions do not have to follow a repeating pattern. Rather, students should focus on figuring out where the pattern starts and then how much to add to get from one number on the list to the next. Remind students to be as clear as possible, because their classmates are going to follow their instructions.

5. Give the pairs time to make their instructions. As they work, walk around the room, helping them with the number patterns and reminding them about qualities that good instructions have. If pairs finish early, they can choose a new number list and try to make instructions for that list's Number Pattern Boogie.

6. When all students are ready, each pair will trade instructions with another pair. If you have time, each pair can perform the Boogie for the entire class. If not, the pairs that traded can perform for each other.

7. Before the end of class, gather students together to discuss. Were they able to make better instructions using the feedback from last time? What was challenging about the activity this time? How are these number patterns and the instructions that they made different from the number patterns and instructions from the pre-activity?

Number Line Tightrope Post-Activity (Continued)

In particular, presuming that one of the pairs selected the triangular numbers to boogie, you can focus the discussion on that family. What do your students notice about the amount you have to add to one triangular number to get the next? And the next? Try to guide the conversation toward the realization that to obtain the triangular numbers, you start with the number one, then add two, then add three, then add four–each time you boogie forward, you go one space further than the last time.

8. Conclude by reminding students that number lines are a great tool for knowing the order numbers go in, for counting up, for counting down, for adding, for subtracting, and for organizing other information we know about numbers.

Extension: Boogie-ing Backward

Try a subtraction Number Pattern Boogie. Start at the highest number on the number line and boogie subtract your way back to 1. For an extra challenge, have students use one of the *Number Line Tightrope* number patterns and try to subtract or count backwards to start from the highest number and work backwards toward the lowest. This challenge may only be appropriate for students later in 1st grade.

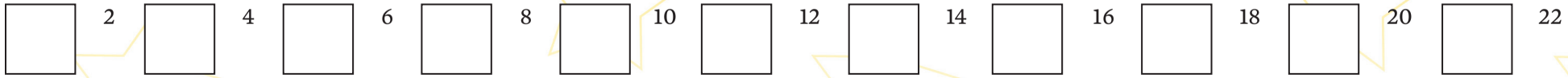
Extension: Number Families

Many of the number families on the *Number Line Tightrope* require prior knowledge that is not expected of 1st graders. However, some of the number families can indeed be taught to 1st graders. Use the attached document *Number Line Tightrope Number Family Explanations and Examples* to help you explain a few number families to your students.

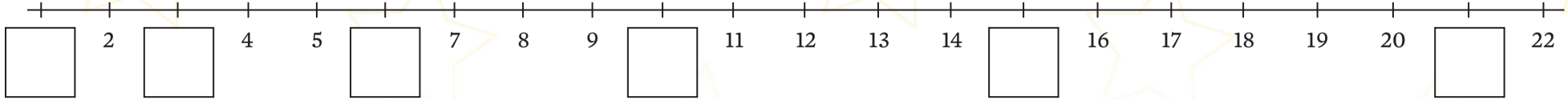
Explore the Number Line!

1. Fill in the missing numbers in the boxes on number lines 1 through 4.

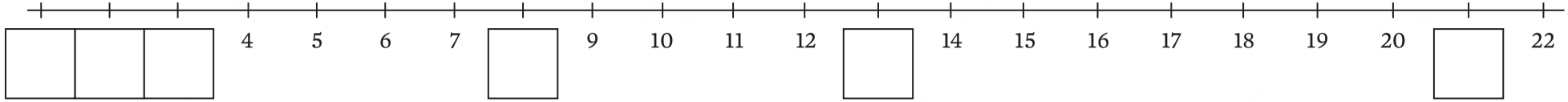
Number Line 1:



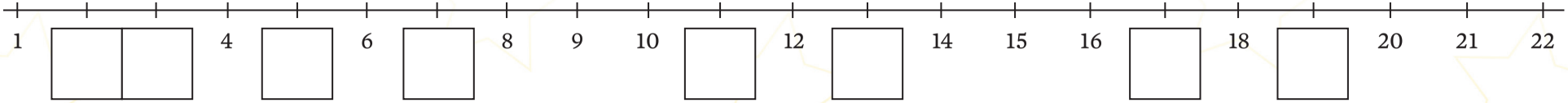
Number Line 2:



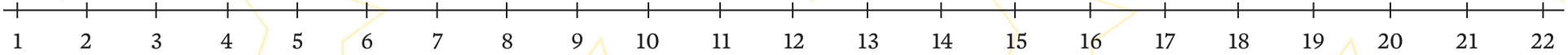
Number Line 3:



Number Line 4:



Number Line 5:



2. Pick a number, any number, on Number Line 5 above, and circle it. Then, skip a number and circle a number. Skip a number and circle a number. Do that going in both directions from your number until you reach the ends of the number line. List all the numbers you circled.

3. Find the smallest number on your list. What do you have to do to that number to get the next number on the list? What do you have to do to THAT number to get the next number on the list?

Number Line Tightrope Recording Sheet

Draw the symbol you see.	Where do you see it? List all of the numbers where this symbol appears.

Number Line Tightrope Number Family Explanations and Examples

Fibonacci Sequence

Leonardo of Pisa, also called Fibonacci, started a list of numbers with 0 and 1. To find the next number, he added the last number to the number before: 1+0=1. The list was then 0, 1, 1. He repeated this process to find the fourth number in the list: 1+1=2. Then the list was 0, 1, 1, 2. Continuing this process generates the Fibonacci sequence.

Fibonacci numbers on the *Number Line Tightrope*: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89
Beyond the Century: What are the next two numbers in the Fibonacci Sequence? 144, 233

Square Numbers

Using square blocks, make squares that start small and then get slightly bigger. The smallest square uses only 1 square tile. The next larger square you can make uses 4 square tiles. The following square uses 9 tiles. Students working in groups can come up with the family of square numbers even before they learn how that relates to multiplication and/or exponents.

0 is also a square number, but this fact may be omitted when working with young students.

Square numbers on the *Number Line Tightrope*: 0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100
Beyond the Century: What are the next two square numbers? 121, 144

Cubes

Students will need prior knowledge either of three-dimensional cubes or of exponents. Start by having students identify the square numbers on the *Number Line Tightrope*. Then point out that the square symbol and this other blue symbol look similar (make sure you don't say "cube" yet–it gives away the answer too soon). Ask students to figure out where this symbol is found and why it might look like the square. At this point, a student who is familiar with either drawing cubes or exponential notation will figure out that the symbol means cubes.

Number Line Tightrope Number Family Explanations and Examples (Continued)

Have students prove it–pick a positive cube, like 27, and ask: what number cubed gives 27? The answer is 3 because 3 * 3 * 3 or 3^3 is 27. Have students find 1^3, 2^3, and 4^3. Then, ask students what 5^3 would be. The answer is 125, which is beyond this number line.

Note that 0 is also a cubed number–help students identify that 0 is the value of 0^3. If students are comfortable with negatives, ask students: are all the cubes positive? The answer is no, since -1 and -8 are cubes. Help students figure out why this is the case: -1 is the cube of -1 and -8 is the cube of -2.

Cubes on the *Number Line Tightrope*: -8, -1, 0, 1, 8, 27, 64
Beyond the Century: What are the next two cubes? 125, 216

Powers of Two

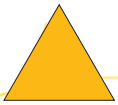
Ask students to find this symbol anywhere on the number line. Then, have them carefully look for the preceding and following instances of the symbol. So, if they found 16, they would look to find 8 before and 32 after. Ask students: what is the relationship between these numbers? If students haven't figured it out, have them keep looking for preceding and following instances of the symbol. At some point, a student will figure it out–the following number is always double and the preceding one is half. Then ask students to find all the instances of this symbol on the *Number Line Tightrope*–where does the pattern start? It starts at 1 and doubles each time to get to 64. The following double, 128, is off the number line. This family of numbers is called Powers of Two.

If students are comfortable with exponents, you can link the name of the family to exponential notation–each of these numbers can be written as two to the power of something, which is to say 2^x. Practice by taking each number with the Powers of Two symbol and figure out what the value of x is for each.

Powers of Two on the *Number Line Tightrope*: 1, 2, 4, 8, 16, 32, 64
Beyond the Century: What are the next two Powers of Two? 128, 256

Number Line Tightrope Number Family Explanations and Examples (Continued)

Triangular Numbers

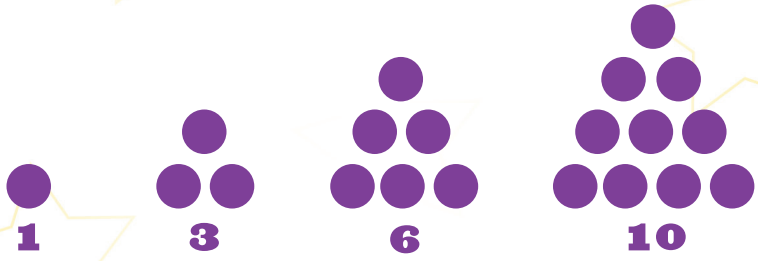


Ask students to start at 0 and then find the following triangular number: 1. Ask them how the number grew. (It went up by 1.) Then find the following triangular number: 3. Ask students how the number grew this time. (It went up by 2.) Repeat this investigation until students see the pattern–each time, you add the following whole number (+1, +2, +3, +4, etc.) So, why are these numbers called triangular numbers? Use counting chips or coins to build triangles. Start with one coin–this could have an equilateral triangle built around it. Then, add a row of two coins below–this now looks like a larger equilateral triangle. For the next row, ask students: how many coins will we add? Have a student demonstrate adding three coins to make a triangle using six coins in total:

Have students build ever-larger equilateral triangles side by side and link these physical triangles to the yellow triangle symbol on the number line. Triangular numbers are the numbers of coins or tiles that can be used to make physical equilateral triangles.

0 is also a triangular number, but it is okay to omit it when working with young students.

Triangular numbers on the *Number Line Tightrope*: 0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91
Beyond the Century: What are the next two triangular numbers? 105, 120



Number Line Tightrope Number Family Explanations and Examples (Continued)

Pizza Numbers

This number family works best if you have a drawing surface and can draw a diagram as you go.

Tell students that they work in a very silly pizza shop. They start with an entire pizza and can cut only straight lines. If they make zero cuts, how many slices will they have? They will have 1 very large slice. Now, they get to make one cut–how many slices now? There are 2. Cut one more time (two cuts) and there are 4 slices. Explain that here is where it gets crazy–how many slices can students make with the third cut? Students will typically say 6, or perhaps 8 (but that requires making two more cuts). Have students draw how they would get to 6–they will draw a third line going through the intersection of the first two lines. Then, remind students that this is a silly pizza shop–unlike a standard pizza shop, students do NOT have to cut through the intersection, or through the center. Ask students: could you go from 4 slices to more than 6 slices with the third cut? Eventually, a student will show with their finger or a writing implement that if you cut through three regions, you can get to 7 slices:

This list (1, 2, 4, 7) is the beginning of the pizza numbers. Ask students–is there a pattern? Yes: like the triangular numbers, the pizza numbers grow each time according to a pattern–each time you add the following whole number (+1, +2, +3, +4, etc.). Use this pattern to predict the next pizza numbers and then verify those guesses by looking at the *Number Line Tightrope*. Of course, these are slices of unequal size, but it is possible to get 92 slices with 13 cuts!

Pizza numbers on the *Number Line Tightrope*: 1, 2, 4, 7, 11, 16, 22, 29, 37, 46, 56, 67, 79, 92
Beyond the Century: What are the next two pizza numbers? 106, 121

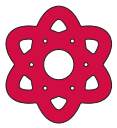
Number Line Tightrope Number Family Explanations and Examples (Continued)

Factorial Numbers

Ask students what an exclamation point means when they are writing. After students relate their thoughts, explain that in math, the exclamation point has a special meaning–it is called a factorial. 3! means to multiply 3 * 2 * 1–what does that equal? The answer, 6, is a factorial number. Ask students to predict–what would 4! be? Have students try to generalize: if 3! starts with 3 and then multiplies it by the smaller whole numbers down to 1, see if students can generate the list 4 * 3 * 2 * 1 for 4!. Then, evaluate: 4! is equal to 24. So, 24 is also a factorial number. Then, have students figure out 1!, 2!, and 5!. Make sure they notice that 1!, 2!, 3!, and 4! are all on the *Number Line Tightrope* while 5!, which is 120, is too high to show up here.

Factorial numbers on the *Number Line Tightrope*: 1, 2, 6, 24
Beyond the Century: What are the next two factorial numbers? 120, 720

Additional Number Families



Prime numbers are the family of numbers with only two factors–the number one and the prime number itself.
Prime numbers on the *Number Line Tightrope*: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
Beyond the Century: What are the next two prime numbers? 101, 103

Highly composite numbers are positive composite numbers that have more factors than any prior positive number on the number line.
Highly composite numbers on the *Number Line Tightrope*: 4, 6, 12, 24, 36, 48, 60
Beyond the Century: What are the next two highly composite numbers? 120, 180

Number Line Tightrope Number Family Explanations and Examples (Continued)

Perfect numbers are numbers whose factors (other than the number itself) sum to the number.

Perfect numbers on the *Number Line Tightrope*: 6, 28

Beyond the Century: What are the next two perfect numbers? 496, 8128

Cake numbers are a three-dimensional version of pizza numbers.

All the cake numbers on the *Number Line Tightrope*: 1, 2, 4, 8, 15, 26, 42, 64, 93

Beyond the Century: What are the next two cake numbers? 130, 176

Constructible polygon numbers are the numbers of edges of the regular polygons that can be constructed using only a compass and a straightedge.

Constructible polygon numbers on the *Number Line Tightrope*: 3, 4, 5, 6, 8, 10, 12, 15, 16, 17, 20, 24, 30, 32, 34, 40, 48, 51, 60, 64, 68, 80, 85, 96

Beyond the Century: What are the next two constructible polygon numbers? 102, 120

Pentagonal numbers are the pentagonal extension of the triangular and square numbers.

Pentagonal numbers on the *Number Line Tightrope*: 1, 5, 12, 22, 35, 51, 70, 92

Beyond the Century: What are the next two pentagonal numbers? 117, 145

Tetrahedral numbers extend triangular numbers into three dimensions, making tetrahedra rather than triangles. Tetrahedral numbers are to triangular numbers as cubes are to square numbers.

Tetrahedral numbers on the *Number Line Tightrope*: 1, 4, 10, 20, 35, 56, 84

Beyond the Century: What are the next two tetrahedral numbers? 120, 165