Math Midway 2GO

Roller Graphicoaster

Lesson Plan

GRADE 2

Brought to you by MoMath

A Letter from the PSEG Foundation

My fascination with energy started at a young age.

The Arab oil embargo of the 1970's sent gasoline prices through the roof and made clear how closely tied our country's foreign policy is to oil interests. I began wondering whether we could produce energy in ways that didn't involve oil, and I wanted to be part of the quest to find the answer.

That passion led me to pursue years of study in the fields of physics and engineering. Graduate degrees in those areas allowed me to take on a variety of fascinating assignments in my career. I served as a research scientist at the Princeton Plasma Physics Lab, a Congressional Science Fellow in the office of U.S. Senator Bill Bradley, and a science, energy, and technology policy advisor to Governor Tom Kean before coming to PSEG where I work every day to create and deliver power responsibly.

This curriculum, developed by the Museum of Mathematics and funded by PSEG, is intended to help young people develop an interest in math and the technical fields–to spark curiosity, stimulate inquiry, and help students down a path of discovery that leads to fulfilling careers.

As issues such as climate change, energy independence, and national security demand increasingly comprehensive and technical solutions, the need for people with knowledge in science, technology, engineering, and math–areas known as the STEM subjects–will continue to grow.

At PSEG, we understand that our country's future depends on developing the insights, creativity, and dynamism of the next generation of innovators. This curriculum is one of many investments we've made in an effort to help young people discover their talents and develop a thirst for knowledge.

A math- and science-savvy workforce will lead the way to innovative technological discovery, a strengthened economy, and thriving new industries. And it is an important part of building a talent pipeline for the energy industry and our country as a whole.

Ralph Izzo
Chairman, CEO and President, PSEG

MoMath is pleased to acknowledge the support of the
Alfred P. Sloan Foundation in the creation of *Math Midway 2 Go*, and the support
of the PSEG Foundation in the creation of the accompanying curriculum.

Alfred P. Sloan Foundation

PSEG Foundation

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General Instructions for *Math Midway 2 Go*

Math Midway 2 Go (MM2GO) consists of six interactive mathematics exhibits that can travel to schools and other venues. Hands-on activities captivate and engage students, highlighting the wonder of mathematics. These exhibits were designed for use with individuals of all ages, and the mathematical topics they address range from topics in the elementary classroom to college-level mathematics. Students of all ages will benefit from open exploration of the exhibits. At the same time, the exhibits also tie into specific curricular topics for kindergarten through grade 12.

These lesson plans are provided by MoMath to support teachers like you. To help you and your students make the most of your time at *Math Midway 2 Go*, a focus exhibit has been selected for each grade from kindergarten though grade 12. The Grade 2 focus exhibit is the *Roller Graphicoaster*.

MM2GO is designed to accommodate one class of up to 36 students at a time.

It is ideal to have only a small group of students at each exhibit while visiting *Math Midway 2 Go*. Break your class into six groups and have them rotate through the exhibits, with one group at each exhibit at a time. Before starting, make sure that students understand basic rules for interacting with the exhibits:

- Walk in the area surrounding the exhibits; don't run.
- Handle the exhibits gently.
- Do not hang or lean on the *Number Line Tightrope*.
- Handle *Ring of Fire* shapes gently.

Ideally, school support staff and/or parent volunteers will be available for the duration of the visit to *Math Midway 2 Go*. These adults can circulate throughout the exhibits, while the classroom teacher remains at the focus exhibit. At the five exhibits that are not the grade-level focus, students can explore and play.

Information about the *Roller Graphicoaster*

About the exhibit:

The *Roller Graphicoaster* is a model roller coaster with an adjustable track. Students attempt to discover the shape of the track that will produce the shortest time for the roller coaster car to slide from a fixed starting point to a fixed ending point. Students can try one of the suggested curves (a straight line, a parabola, a cubic curve, a sine curve, a circular arc, or a cycloid), or design their own track shape from scratch. Older students can use calculus to analyze the problem, while younger students can focus on intuition and experimentation to figure out which qualities make the fastest coaster.

Why visit the *Roller Graphicoaster*?

The *Roller Graphicoaster* is based on a math challenge first posted by Galileo Galilei in the 17th Century. He wondered: what is the fastest path from one point to another when gravity is involved? Galileo was unable to solve the problem, and it remained unsolved until after the invention of calculus 60 years later.

With elementary school students, exploring the *Roller Graphicoaster* is a chance to question assumptions–many students already know that the shortest distance between two points is a straight line and they will equate a shorter distance with a shorter time. Is this correct? In fact, due to the effect of gravity, a straight-line roller coaster is not the fastest possible roller coaster. When using the *Roller Graphicoaster*, students will make predictions and test them before ultimately rejecting "short" as the quality of a roller coaster that optimizes time, in favor of other qualities. In the pre-activity and post-activity, students will practice using rulers, measuring tape, and other tools to measure straight and curved paths, solidifying their early elementary study of measurement. Many students have never been asked to measure a curved path before, but they will be able to use their mathematical reasoning to try out different ways to accomplish the task, ultimately coming up with a method that allows for measuring curves and curved surfaces.

Integrating MM2GO Into Your Unit Plans

Consider the following key questions, class topics, and elements of the Common Core State Standards when considering how to link the *Roller Graphicoaster* to the study of mathematics taking place in your classroom.

Key questions inspired by the *Roller Graphicoaster*:

- Is there a way to measure a path that is not straight?
- What tool is appropriate for measuring a straight line? What tool is appropriate for measuring a curve?
- What is the shortest distance between two points? What happens when gravity is included–is the shortest path always the fastest path?
- When designing a fast path, what qualities should it have? For example, should it be steep or gently sloping? What other qualities might matter?

This lesson plan will be useful with the following classes:

- Classes studying measurement and practicing using rulers and measuring tape
- Classes learning about measuring curved surfaces or curved paths for the first time
- Classes studying gravity and the role it plays with respect to moving objects

Relevant connections to the Common Core State Standards:

Learning Standards

2.MD: Measure and estimate lengths in standard units.

Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Use appropriate tools strategically.
- Attend to precision.

Roller Graphicoaster Pre-Activity

Description

In this activity, students will measure various paths to prepare them for measuring the length of their roller coaster track when they visit the *Roller Graphicoaster*.

While this activity is designed for use before visiting the *Roller Graphicoaster*, the activity can be enjoyed independently of a visit from the Museum of Mathematics' *Math Midway 2 Go*.

Materials

- Rulers
- Measuring tape
- String
- Small straight objects like paper clips or toothpicks
- Attached *Measuring a Path* worksheet

Key Terminology

- **Line**
- **Line segment**
- **Straight**
- **Curved**
- Units of measurement including **inches** and **centimeters** as well as non-standard units of measurement

Conducting the Activity

1. Group students in pairs.

2. Hand out the *Measuring a Path* worksheet to each student. Explain to students that they need to find the length of the four paths they see. Ask them to work with their partner to make a plan for how they will measure the different paths the ant takes to get to his food.

3. Show students the materials you have prepared: rulers, measuring tape, string, and the small straight objects. Students may also use other materials in the room if they have a different plan.

4. Give students time to make a plan with their partner for measuring each line. Then, have a discussion with the whole class. Ask students to share how they are going to measure the four paths. Discuss which path is simplest to measure, which one is hardest to measure, and what tools they will need to measure each of the paths.

Roller Graphicoaster Pre-Activity (Continued)

Your students should be comfortable with measuring the completely straight path- Path A. The second path, Path B, is a series of line segments–help students figure out that they can measure this path by measuring each segment like they would measure a line and then adding the measurements to find the total distance the ant has to walk. The curved line paths, Paths C and D, will be hardest, as measuring them may involve new ideas the students have never encountered before.

Some students may suggest using measuring tape and twisting it in line with the path.

Students could also use lengths of string and twist them to fit the path, then stretch the string along a ruler to measure the path.

Allow students to try whatever method they suggest. If students use non-standard measuring tools, like toothpicks, they can lists length in that unit. Some of the paths may require fractional measurements. You can either have students round the measurements, or support them in recording fractional amounts.

5. Then, pass out rulers, measuring tape, and string to each pair of students. Have them figure out the length of each path, and record that length on the worksheet.

6. When a pair of students is finished, instruct those students to find another pair and share their results. Compare the measurements–how similar are they? If the results are very dissimilar, how can students figure out which measurement is more accurate? Encourage students to measure again and check their work if the other pair's answer is very different.

7. Once all students have completed their measurements, make a class table that lists all the results students got for each path. Based on these many measurements, what is the approximate length of Path A, Path B, Path C, and Path D?

Roller Graphicoaster Pre-Activity (Continued)

8. It is likely that students will mostly agree on the lengths of Paths A and B. If there are large discrepancies among the measured lengths for Paths C or D, discuss the problem–what makes Paths C and D so difficult to measure?

9. Finish the class by explaining that students will be visiting *Math Midway 2 Go* and exploring how to design a fast roller coaster with the *Roller Graphicoaster*. Their new skills in measuring curved paths will come in handy for measuring the curved track of the roller coaster.

Extension: Straight and Curved Measurement

Find other curved surfaces around the classroom and practice measuring their length. Students can use the method of their choice to determine length. Have students record their results – what they measured, its length, and the method they used to make the measurement.

Extension: How Long is a Toothpick?

Some of your students used non-standard units of measurement, like toothpicks or paper clips, to measure curvy paths. Can students change those non-standard measurements to standard units of measurement? Work with students to come up with a method. You might measure one paper clip and then make a chart that adds up each paper clip, so students know how long their curvy path is in inches or centimeters.

Roller Graphicoaster Activity

Description

In this activity, students will explore the *Roller Graphicoaster*, making predictions about which track makes the fastest coaster, and then testing those predictions.

Materials

- Attached *Roller Graphicoaster Observation Sheet*, one copy per student
- Pencils
- Optional: clipboards

Key Terminology

- **Line**
- **Line segment**
- **Straight**
- **Curved**
- **Steep**
- **Flat**
- **Shallow**
- **Fast**
- **Slow**

Conducting the Activity

1. Start by having students examine the colored lines in the background of the *Roller Graphicoaster*. Each colored line is a suggested path that the *Roller Graphicoaster* can take. Ask students–which path is shortest? Ask them to predict which path will be the faster roller coaster track. Will the shortest be the fastest? What other factors might lead to a fast track?

2. To test their predictions, have each student in the group pick one of the suggested paths and adjust the posts of the *Roller Graphicoaster* to match that path. Make sure all of the paths will be tested.

3. One at a time, have each student try their assigned path–use the sliders to move the brackets up and down. The track can be pulled left or right to add or remove slack as needed. While one student adjusts the track, other students can discuss their predictions for this run. They should also sketch the track on their observation sheet–discuss what makes an accurate sketch.

Roller Graphicoaster Activity (Continued)

4. Once the track is ready, have the student press the button labeled “Push Gently.” It is important to push gently–the clock is most accurate when students press slowly.

5. After each run, have students record the time elapsed. Discuss–was the track faster or slower than expected? Why might that be?

6. When students have results from all six tracks, allow them to experiment freely- what other tracks can they design?

7. Later, when all the groups have had a turn at the *Roller Graphicoaster*, have a whole class discussion. What happened at the *Roller Graphicoaster*? What was expected? What was unexpected? What qualities made the fastest roller coaster?

8. Explain to students that they will be examining what makes a faster coaster in more detail when they return to the classroom.

Roller Graphicoaster Post-Activity

Description

In this activity, students will measure the paths of the Roller Graphicoaster that they tested during their visit to Math Midway 2 Go, to determine the relationship between length and time.

This activity is designed for use after visiting the Roller Graphicoaster, and requires student data from visiting the exhibit.

Materials

- Rulers
- Measuring tape
- String
- Small straight objects like paper clips or toothpicks
- Attached *Roller Graphicoaster Observation Sheet* (The same copies students used during their visit to the MM2GO should be used again.)
- Attached *Roller Graphicoaster Drawing*, one copy per student

Key Terminology

- **Line**
- **Line segment**
- **Straight**
- **Curved**
- **Steep**
- **Flat**
- **Shallow**
- **Hill**
- **Valley**
- Units of measurement including **inches** and **centimeters**

Conducting the Activity

1. Ask students: what did you discover during your exploration of the *Roller Graphicoaster*? What do you remember?

2. Ask students to consult their data. Which roller coaster was fastest? Ask students: why do you think this roller coaster was fastest?

3. Explain to students that today they are going to measure the distance each roller coaster track traveled. When the roller coaster travels in a curved path, students will use their new methods for measuring curves to find the distance.

Roller Graphicoaster Post-Activity (Continued)

4. Pass out the *Roller Graphicoaster Drawing*.

5. Split students into six groups. Assign each group one of the six tracks. Each student in the group will measure the group's assigned track using whatever method s/he chooses–measuring tape, ruler, string, or other non-traditional measuring material. Make sure students see the measuring tools they have at their disposal.

6. Once all students in a given group have a measurement for their track, compare results. Are the lengths that students measured the same or different? Why might the results be slightly different? Reassure students that when measuring a curved path with straight tools, a small amount of difference between one person's answer and another's is normal–as long as each student's numbers are not very different from those found by their classmates, they should trust their measurements.

7. If other groups are still working, students who finish early can measure the length of another curve.

8. Once all groups have at least one curve measured, have students stand up. When you say go, students will walk around the room and look for a partner who has measured a different curve. The two students will trade results and then each will move on, seeking a new partner with yet another different curve, until all students have data on all six curves. Students who are finished will show they are finished by returning to their seats and sitting down.

9. When all students understand, tell them "Go!"
   If some students are left over at the end when many others have been seated, make sure the standing students understand that they can trade data with seated students, too.

Roller Graphicoaster Post-Activity (Continued)

10. Now that students have results for the length of each track and the time it takes for the roller coaster to zip along each track–discuss as a class. Is the track that takes the least amount of time the shortest? Is it the longest? If length is not the quality that allows one track to take less time than another, what is?

11. Students may bring up the word **steep** in their discussion. As a class, define the word steep. Use hand motions to show a steep track and contrast it with a flat track. Then explain to students that steepness is certainly important, but that it is not the only factor in making a fast track.

    - On the board, draw for students a track that is very flat for the first half of its length and very steep for the second. Then, draw a track that is very steep for the first half of its length and very flat for the second. Which do they think will be faster? Why? Their instinct will likely lean toward the second track–which is correct, because the sooner a track allows the roller coaster to speed up under the influence of gravity, the less time the roller coaster will take to make it from start to finish. **Therefore, which section of the track is steep (near the beginning or near the end) matters.**

    - Now, draw a track that is a straight line from the Start point to a point well below the Finish point and roughly half the horizontal distance between Start and Finish, and then a straight line back up to the Finish. This track is very steep, but it has a problem–ask students to predict what would happen with this track. Indeed, the roller coaster would have trouble climbing the uphill portion of the track and might get stuck in the valley. **So avoiding sharp valleys is also an important part of roller coaster design.**

    - Even if the track has no valleys, imagine a track that is flat for a portion of its length. What would happen then? Ask students to explain how a flat track would affect the speed of the roller coaster. Draw two versions of

Roller Graphicoaster Post-Activity (Continued)

similar tracks–one with a shape that might go fast and one that looks similar but starts flat. As students will deduce, a flat track does not allow the roller coaster to speed up, and adds time to the roller coaster's run. **Avoiding flat sections in general, whether valleys or plateaus, makes for faster roller coaster design.**

12. This is why the Cycloid Cyclone is so fast–it dips down quickly, avoids flat sections, and never has a sharp valley where the roller coaster could get stuck.

13. End by explaining to students that this problem of making the fastest roller coaster connecting two points was posed by Galileo Galilei in 1638, and that he could not figure out the best answer. It took the development of calculus to really design the cycloid curve and that students can study calculus in high school, when they will learn more about what makes this roller coaster so fast.

Extension

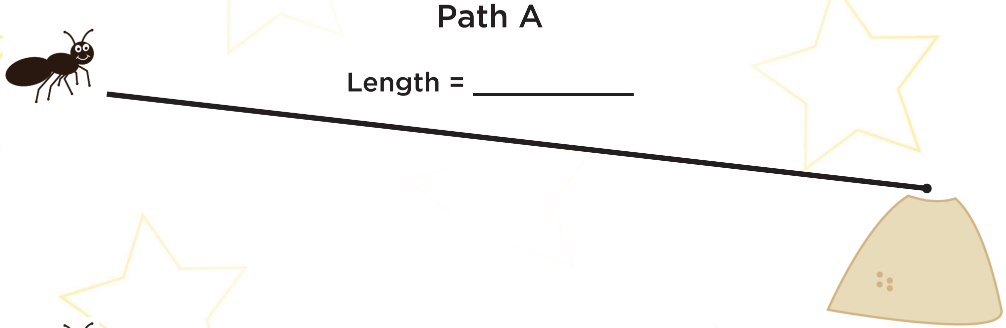
Can you make a *Roller Graphicoaster* in your classroom? Using model race car tracks or some other flexible, flat material, have students test a roller coaster track with a marble. Try to design a track that uses some of the important qualities that students think will make a fast roller coaster, and see what happens.

Measuring a Path

The ant needs to get home to his anthill. How far away is his home? Measure each path. Make sure you include a unit with your answer!

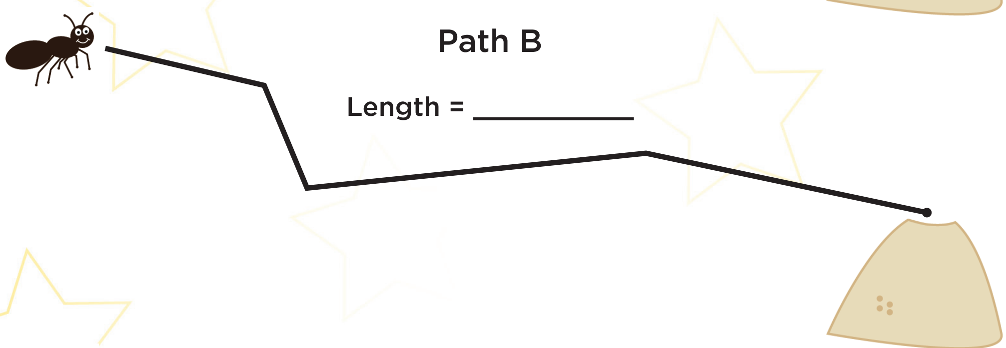
Path A

Length = ___________



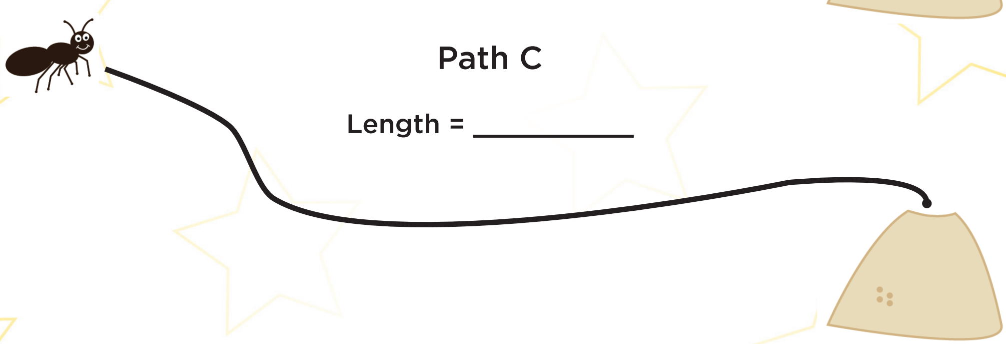
Path B

Length = ___________



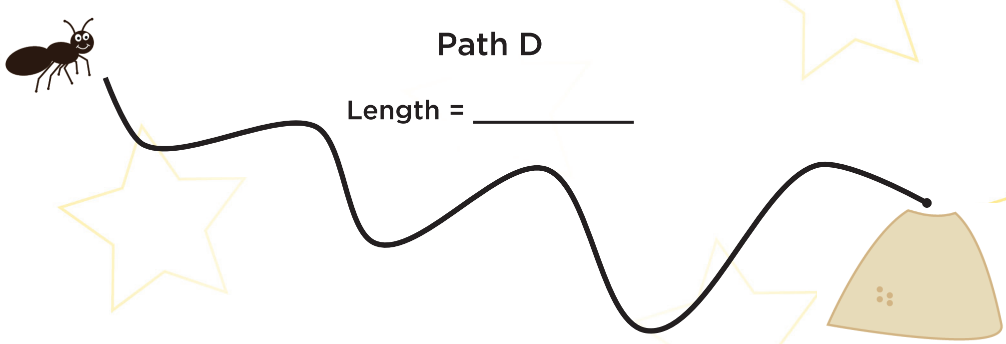
Path C

Length = ___________



Path D

Length = ___________

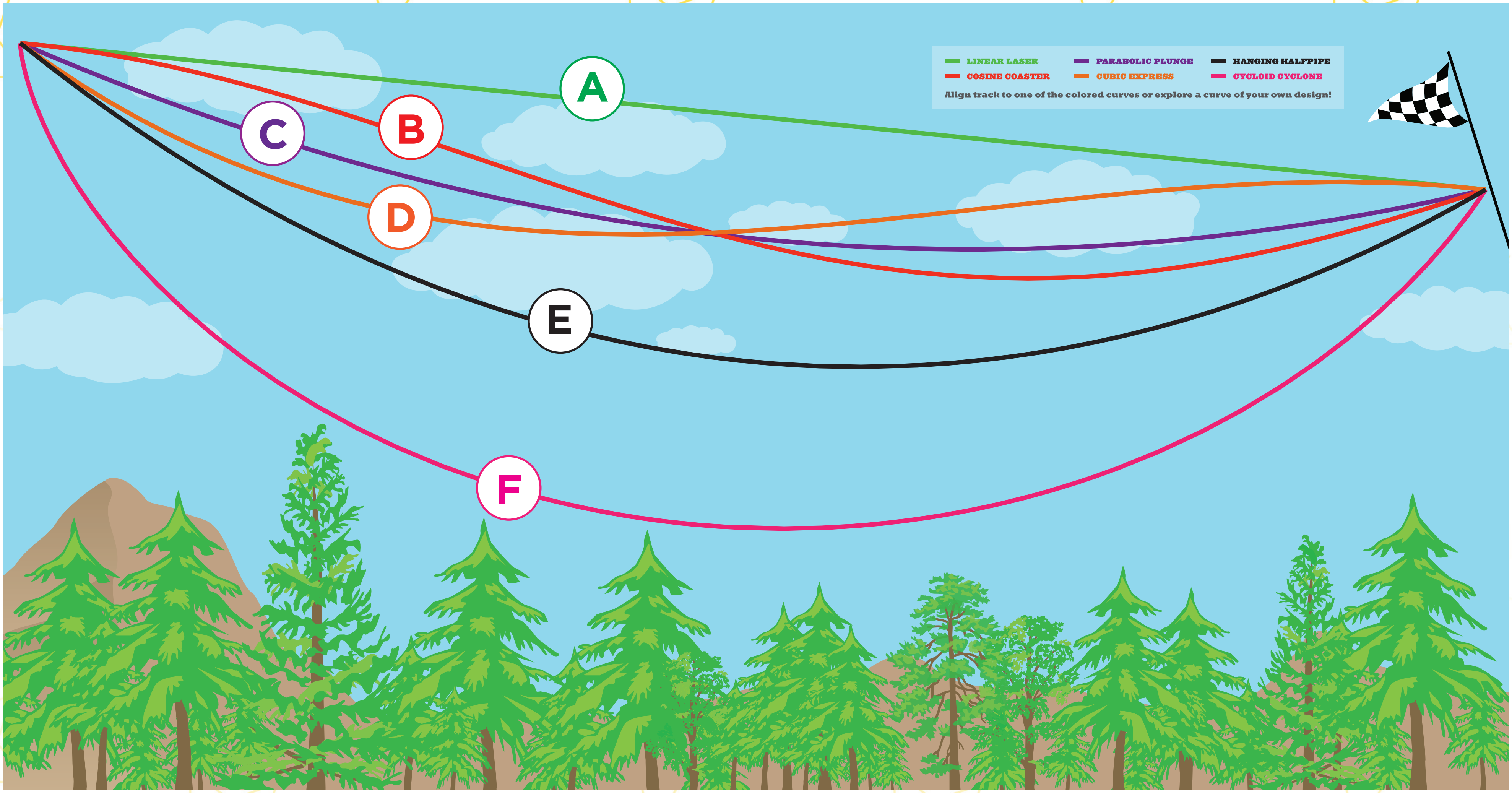


Roller Graphicoaster Observation Sheet

How fast is each roller coaster? Record your results. Save this paper, since you will need it when you measure the track's length back in your classroom.

Track Name	Sketch it!	Time	Length
Cosine Coaster			
Cubic Express			
Cycloid Cyclone			
Hanging Halfpipe			
Linear Laser			
Parabolic Plunge			

Roller Graphicoaster Drawing



Key

- A Linear Laser
- B Cosine Coaster
- C Parabolic Plunge
- D Cubic Express
- E Hanging Halfpipe
- F Cycloid Cyclone

MATH MIDWAY 2GO