MATH MIDWAY 2GO

MILES of TILES

Lesson Plan

GRADE 3

Brought to you by MO MATH

A Letter from the PSEG Foundation

My fascination with energy started at a young age.

The Arab oil embargo of the 1970's sent gasoline prices through the roof and made clear how closely tied our country's foreign policy is to oil interests. I began wondering whether we could produce energy in ways that didn't involve oil, and I wanted to be part of the quest to find the answer.

That passion led me to pursue years of study in the fields of physics and engineering. Graduate degrees in those areas allowed me to take on a variety of fascinating assignments in my career. I served as a research scientist at the Princeton Plasma Physics Lab, a Congressional Science Fellow in the office of U.S. Senator Bill Bradley, and a science, energy, and technology policy advisor to Governor Tom Kean before coming to PSEG where I work every day to create and deliver power responsibly.

This curriculum, developed by the Museum of Mathematics and funded by PSEG, is intended to help young people develop an interest in math and the technical fields–to spark curiosity, stimulate inquiry, and help students down a path of discovery that leads to fulfilling careers.

As issues such as climate change, energy independence, and national security demand increasingly comprehensive and technical solutions, the need for people with knowledge in science, technology, engineering, and math–areas known as the STEM subjects–will continue to grow.

At PSEG, we understand that our country's future depends on developing the insights, creativity, and dynamism of the next generation of innovators. This curriculum is one of many investments we've made in an effort to help young people discover their talents and develop a thirst for knowledge.

A math- and science-savvy workforce will lead the way to innovative technological discovery, a strengthened economy, and thriving new industries. And it is an important part of building a talent pipeline for the energy industry and our country as a whole.

Ralph Izzo
Chairman, CEO and President, PSEG

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Alfred P. Sloan Foundation

PSEG Foundation

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General Instructions for *Math Midway 2 Go*

Math Midway 2 Go (MM2GO) consists of six interactive mathematics exhibits that can travel to schools and other venues. Hands-on activities captivate and engage students, highlighting the wonder of mathematics. These exhibits were designed for use with individuals of all ages, and the mathematical topics they address range from topics in the elementary classroom to college-level mathematics. Students of all ages will benefit from open exploration of the exhibits. At the same time, the exhibits also tie into specific curricular topics for kindergarten through grade 12.

These lesson plans are provided by MoMath to support teachers like you. To help you and your students make the most of your time at *Math Midway 2 Go*, a focus exhibit has been selected for each grade from kindergarten though grade 12. The Grade 3 focus exhibit is *Miles of Tiles*.

MM2GO is designed to accommodate one class of up to 36 students at a time.

It is ideal to have only a small group of students at each exhibit while visiting *Math Midway 2 Go*. Break your class into six groups and have them rotate through the exhibits, with one group at each exhibit at a time. Before starting, make sure that students understand basic rules for interacting with the exhibits:

- Walk in the area surrounding the exhibits; don't run.
- Handle the exhibits gently.
- Do not hang or lean on the *Number Line Tightrope*.
- Handle *Ring of Fire* shapes gently.

Ideally, school support staff and/or parent volunteers will be available for the duration of the visit to *Math Midway 2 Go*. These adults can circulate throughout the exhibits, while the classroom teacher remains at the focus exhibit. At the five exhibits that are not the grade-level focus, students can explore and play.

Information about *Miles of Tiles*

About the exhibit:

Miles of Tiles is a magnet wall with various magnetic tile shapes–triangles, squares, hexagons, two types of rhombuses, and monkeys. As students play, they can explore many aspects of shape and pattern, observing how various tiles fit together. Note that *Miles of Tiles* is designed in particular to help students explore **tessellations**: patterns of tiles with no gaps and no overlaps, that could extend to cover the entire plane. Each of the shapes tessellates by itself, including the irregularly shaped monkeys, which most people find surprising. Some shapes will also tessellate in combination with other shapes (rhombus and square), whereas other combinations of shapes (square and hexagon) do not tessellate.

Why visit *Miles of Tiles*?

Miles of Tiles is a magnetic wall accompanied by magnetic shapes–triangles, squares, rhombuses, hexagons, and monkeys–that students can use to create all kinds of patterns and designs. The exhibit is intuitive–people of all ages are drawn to it–and lends itself to great discussions about many aspects of shapes.

Elementary school students are learning about fractions in many forms. One way to explore fractions is through dissecting shapes into identical pieces, with tools like pattern blocks and the *Miles of Tiles* magnets. Through hands-on exploration, students will gain a deeper understanding of the parts of a whole.

In the pre-activity, students will practice covering one pattern block shape with others to answer questions like “what fraction of a hexagon is a triangle?” or “ What fraction of a hexagon is a trapezoid?” Later, students will make larger designs and practice splitting them into equally sized parts to gain additional perspective on fractions. Through these explorations, students will practice fraction language as both parts of a whole and parts of a group.

Integrating MM2GO Into Your Unit Plans

Consider the following key questions, class topics, and elements of the Common Core State Standards when considering how to link *Miles of Tiles* to the study of mathematics taking place in your classroom.

Key questions inspired by *Miles of Tiles*:

- How can one shape be split into smaller shapes?
- What fraction of a bigger object is one particular part?
- What fraction of a group of shapes is an individual shape?
- Do parts have to be of equal size to use fraction words to describe the situation accurately?

This lesson plan will be useful with the following classes:

- Classes learning fraction terminology
- Classes studying fractions as parts of a whole and parts of a group
- Classes studying how shapes and fractions can relate

Relevant connections to the Common Core State Standards:

Learning Standards

3.NF: Develop an understanding of fractions as numbers.

Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

Miles of Tiles Pre-Activity

Description

In this activity, students will experiment with pattern blocks to discover fractional equivalency.

While this activity is designed for use before visiting *Miles of Tiles*, the activity can be enjoyed independently of a visit from the Museum of Mathematics' *Math Midway 2 Go*.

Materials

- Attached *What Fraction Is It?* worksheet
- Pattern blocks: a mix of the triangles, hexagons, trapezoids, and blue rhombuses

If you do not have access to pattern blocks, you can print the attached template on card stock. These paper pattern blocks were created using NCTM's Dynamic Paper applet http://illuminations.nctm.org/ActivityDetail.aspx?ID=205 *which allows teachers to create their own graphics.*

Key Terminology

- **Triangle**
- **Rhombus**
- **Trapezoid**
- **Hexagon**
- **Fraction** and fraction notation including **numerator** and **denominator**

Conducting the Activity

1. Explain to students that any time you split a whole into parts, the sizes of the parts can be represented with fractions. Today, students will be splitting pattern blocks into smaller pieces without cutting anything.

2. Pass out the pattern blocks–each student should have access to at least six of each shape.

3. Ask students to identify which shapes they have in front of them. Explain that they have the triangle, hexagon, trapezoid, and blue rhombus for a reason–these shapes have a special relationship.

Miles of Tiles Pre-Activity (Continued)

4. Now, have students pick two of these shapes and use one shape to cover the other shape entirely. Make sure that no students end up with the trapezoid/blue rhombus combination.

   If students are struggling, model the task for them with the blue rhombus and the triangle. Place two triangles on top of the blue rhombus to cover it completely.

5. When students have successfully completed this task once, have them share with a partner. Did the students use the same two shapes, one shape that was the same and one that was different, or did they use two entirely different shapes?

6. Then, challenge students to find as many combinations of two shapes as they can, such that a whole number of the first shape completely covers the second shape. How many such combinations can students find?

   For your reference, there are five ways to cover one of these pattern blocks with copies of another:

   - *Two trapezoids cover a hexagon.*
   - *Three blue rhombuses cover a hexagon.*
   - *Six triangles cover a hexagon.*
   - *Three triangles cover a trapezoid.*
   - *Two triangles cover a blue rhombus.*

7. When students have had sufficient time to explore, pass out the *What Fraction Is It?* worksheet. Students will use this to record their discoveries.

8. Pick one example with which to start. For example, you could start with the two trapezoids covering the hexagon. Ask every student in the class to make this combination using the tiles in front of them.

Miles of Tiles Pre-Activity (Continued)

9. Discuss with the class–what part of the hexagon is a trapezoid? What does covering the hexagon with trapezoids show? How many fit? Help students come to the conclusion that the trapezoid splits the hexagon into two equal parts, and that one trapezoid is half a hexagon. Have students imagine taking the whole hexagon and sawing it in half. Make sure they understand that if they followed the seam between the two trapezoids on top, each half of the hexagon would be the same size and shape as the trapezoid they see in front of them.

   The idea that each of the parts has to be of equal size for this fractional language to be accurate is key here. Ensure that students come to the conclusion that when splitting the hexagon, they have to split it into parts of equal size. Pattern blocks make this easy, because we know that two red trapezoids are of equal size–they cover each other perfectly, proving that they are equivalent and therefore that they are of equal size.

10. Once the class has come to the conclusion that a trapezoid is half of a hexagon, help students record that on the *What Fraction Is It?* worksheet. This information will go on the hexagon line.

    In the left column, students will trace a complete hexagon.

    In the middle column, students will trace the hexagon again and then trace the trapezoids to show where the split is.

    In the right column, students will record what fraction this represents. Depending on your preference, this can be a word sentence or a number sentence. For example:

    A trapezoid is half of a hexagon OR 1 trapezoid = ½ hexagon

Miles of Tiles Pre-Activity (Continued)

11. After completing this one example as a class, have students fill in the other four lines on their chart independently.

12. Have students compare their completed charts with a partner–did they remember all five combinations they found? Are the sketches and explanations sufficient to explain the activity to someone who was not present?

13. At the end of the class, explain that students will continue to use magnets to explore fractions and shapes during their visit to the exhibit *Miles of Tiles* at *Math Midway 2 Go*.

Extension: Exploration of Splitting

Students will need access to all four shapes with which they have been working-hexagon, trapezoid, blue rhombus, and triangle. This time, students will cover individual shapes with a mix of shapes. Ask students–is there a way to cover the trapezoid using a mix of shapes, rather than only triangles? There is–students may discover that one blue rhombus and one triangle will cover the trapezoid. Then ask, does this mean that the triangle is a half a trapezoid? Is the blue rhombus half a trapezoid? This discussion will allow students to really explore what it means to split into parts of equal size. Ask students to cover other shapes with a mix of shapes if possible. Then, have a general discussion: does a mix of shapes allow for fractional parts?

As a further activity, allow students to take the shapes that cannot be split into pieces congruent to other pattern blocks–the white rhombus, the square, and the triangle–and have students trace these shapes and sketch where they would cut them to split them into equal parts if they could.

Miles of Tiles Activity

Description

In this activity, students will explore *Miles of Tiles* and identify fractions as parts of groups as well as parts of a whole.

Materials

- There are no specialized materials required for this activity, other than the exhibit itself.

Key Terminology

- **Triangle**
- **Rhombus**
- **Trapezoid**
- **Hexagon**
- **Fraction**

Conducting the Activity

1. Allow students to explore the exhibit at their own pace first, using the magnets to make designs of their choice.

2. Now, ask students to pick a shape they would like to use to create an original design. They can pick any of the tile shapes, including the monkey.

3. Give them one minute to make a design using only that one shape. The design can be as large or as small as they want it to be, given the time allotted.

4. Once students have completed their designs, explain that sometimes fractions come from splitting a complete shape into smaller parts, but sometimes fractions come from parts of a group.

5. Here, students have created a design made of multiple identical parts. Ask students: "In your design, are all the parts of equal shape and size?" Assuming that all students have followed the instructions, they should all answer affirmatively.

Miles of Tiles Activity (Continued)

6. Have students count the total number of shapes they used in their designs.

7. Now ask: "What fraction of your whole design is one shape?" Help students understand that their design's total number of pieces is the denominator of the fraction that answers the question, and that this is true because all the pieces in their design are of equal size.

8. Have students share their designs with each other and establish the answer to the question "What fraction of your whole design is one shape?" for each design they examine. Did any students use the same shape? Is the red rhombus always ⅛ of a design? No–it will depend on how many tiles were used in each student's design.

9. Take some time to explore the blue rhombus that is part of *Miles of Tiles*–ask each student to hold one. Then ask them to predict how many green triangles will cover the blue rhombus. Students will likely assume two, based on the pre-activity. Ask students to test their prediction. In fact, this blue rhombus is different from the blue pattern block rhombus. Explain to students that while the two look similar, they have different angle measurements. The green triangle in the *Miles of Tiles* exhibit is not ½ of the blue rhombus. Now, check the hexagon and the triangle-these will work: with the *Miles of Tiles* magnets, as with the pattern blocks, one triangle is ⅙ of a hexagon.

10. With any remaining time, allow students to explore *Miles of Tiles*, making any designs they choose to make.

11. End by explaining to students that while there is no *Miles of Tiles* magnet wall in their classroom, they will be making more designs out of shapes in their next class, again using pattern blocks.

Miles of Tiles Post-Activity

Description

In this activity, students create a design using pattern blocks and break it down into fractional parts.

While this activity is designed for use after visiting *Miles of Tiles*, the activity can be performed with students who have not had the opportunity to experience the Museum of Mathematics' *Math Midway 2 Go*.

Materials

- Pattern blocks: a mix of the triangles, hexagons, trapezoids, and blue rhombuses

If you do not have access to pattern blocks, you can print the attached template on card stock. These paper pattern blocks were created using NCTM's Dynamic Paper applet http://illuminations.nctm.org/ActivityDetail.aspx?ID=205 *which allows teachers to create their own graphics.*

Key Terminology

- **Triangle**
- **Rhombus**
- **Trapezoid**
- **Hexagon**
- **Fraction** and fraction notation including **numerator** and **denominator**

Conducting the Activity

1. Distribute the mixed pattern blocks to students. Explain that they can make any design they want as long as they only use these four shapes, and they make sure there are no gaps in their design.

2. Allow students to work for a set period of time, about five minutes, on their designs.

3. When five minutes have elapsed, ask students–could you cover the entire design you have made with triangles? Would they fit perfectly on top? Students should come to the correct conclusion that you could cover the entire design with triangles, as we know that triangles can be used to perfectly cover the hexagon, trapezoid, blue rhombus, and the triangle itself.

Miles of Tiles Post-Activity (Continued)

4. Now ask students–how many triangles would it take to cover your design? Students may start to use the remaining green triangles to do this, but they will likely run out of triangles before covering their design. Have students come up with another method to count.

   One method might be to count each shape in their design separately and make a list–how many hexagons they used, how many trapezoids, how many rhombuses, and how many triangles. They know how many triangles are needed to cover one hexagon and can use their multiplication skills to figure out how many are needed to cover multiple hexagons. Indeed, this would be a good time to reinforce mental multiplication or skip counting.

5. Once students have a number, bring this back to fractions. Based on their total-what part of the overall area of their design is one triangle? Each student will have a different answer with one as their numerator and the total as their denominator.

6. Now, ask students if they could cover their designs with a different shape–could they cover it all with trapezoids or rhombuses or hexagons? This will be harder for students, since they may or may not be able to do the task depending on how their design fits together, and on how many triangle units it contains. Ask students to use one different shape (either trapezoids, rhombuses, or hexagons) to see what happens when they try to cover their design. Can the design be covered exactly by pattern blocks in one of these three shapes? Is there any part of the design that remains uncovered, or any part of the covering shape that goes past the edge of the design?

   Here are some math points you can reinforce with your class at this point. One is divisibility–if we know that hexagons are made up of six triangles, could I cover a shape of seven triangles with only hexagons? No–it's not divisible by six. Alternately, you could focus on equal size of parts, making sure that none of the shapes being used to cover extend and hang over the edge the original design. If they do, the original design and the covering are not of equal area.

Miles of Tiles Post-Activity (Continued)

7. With the time remaining, allow students to test their ideas about all of the shapes until they are certain which shapes will and will not cover their design. For any shape that completely covers their design without going past the edge, have students express what fraction of their design one copy of that shape represents.

Extension: Fun with Pattern Block Fractions

The NCTM has more activities for exploring pattern blocks and fractions in their *Fun with Pattern Block Fractions* unit. You can find these activities on their website: http://illuminations.nctm.org/LessonDetail.aspx?ID=U113

Extension: Fun with Tangrams

Like the pattern block fractions, tangrams are a way to explore shapes and fractions. Drexel University's *Math Forum* hosts a lesson unit in which students learn how to make their own tangrams, and then explore the mathematics behind them. The unit is focused on congruence and similarity, but it could also be focused on shapes and their attributes, on area, or on fractions. Find the lesson ideas here: http://mathforum.org/trscavo/tangrams.html

What Fraction Is It?

Use your pattern blocks to explore splitting whole shapes into smaller pieces. Record your explorations here.

Shape Name Trace the shape in the space provided	**Shape Split** What can you split it into? Draw your discoveries in the space provided	**Fraction** What fraction did you find?
Hexagon		
Hexagon		
Hexagon		
Blue Rhombus		
Trapezoid		

Paper Pattern Blocks

Name _____