MATH MIDWAY 2GO

MILES of TILES

Lesson Plan

GRADE

4

Brought to you by MOMATH

A Letter from the PSEG Foundation

My fascination with energy started at a young age.

The Arab oil embargo of the 1970's sent gasoline prices through the roof and made clear how closely tied our country's foreign policy is to oil interests. I began wondering whether we could produce energy in ways that didn't involve oil, and I wanted to be part of the quest to find the answer.

That passion led me to pursue years of study in the fields of physics and engineering. Graduate degrees in those areas allowed me to take on a variety of fascinating assignments in my career. I served as a research scientist at the Princeton Plasma Physics Lab, a Congressional Science Fellow in the office of U.S. Senator Bill Bradley, and a science, energy, and technology policy advisor to Governor Tom Kean before coming to PSEG where I work every day to create and deliver power responsibly.

This curriculum, developed by the Museum of Mathematics and funded by PSEG, is intended to help young people develop an interest in math and the technical fields–to spark curiosity, stimulate inquiry, and help students down a path of discovery that leads to fulfilling careers.

As issues such as climate change, energy independence, and national security demand increasingly comprehensive and technical solutions, the need for people with knowledge in science, technology, engineering, and math–areas known as the STEM subjects–will continue to grow.

At PSEG, we understand that our country's future depends on developing the insights, creativity, and dynamism of the next generation of innovators. This curriculum is one of many investments we've made in an effort to help young people discover their talents and develop a thirst for knowledge.

A math- and science-savvy workforce will lead the way to innovative technological discovery, a strengthened economy, and thriving new industries. And it is an important part of building a talent pipeline for the energy industry and our country as a whole.

Ralph Izzo
Chairman, CEO and President, PSEG

MoMath is pleased to acknowledge the support of the
Alfred P. Sloan Foundation in the creation of *Math Midway 2 Go*, and the support
of the PSEG Foundation in the creation of the accompanying curriculum.

Alfred P. Sloan Foundation

PSEG Foundation

Table of Contents

General Instructions for *Math Midway 2 Go*

Math Midway 2 Go (MM2GO) consists of six interactive mathematics exhibits that can travel to schools and other venues. Hands-on activities captivate and engage students, highlighting the wonder of mathematics. These exhibits were designed for use with individuals of all ages, and the mathematical topics they address range from topics in the elementary classroom to college-level mathematics. Students of all ages will benefit from open exploration of the exhibits. At the same time, the exhibits also tie into specific curricular topics for kindergarten through grade 12.

These lesson plans are provided by MoMath to support teachers like you. To help you and your students make the most of your time at *Math Midway 2 Go*, a focus exhibit has been selected for each grade from kindergarten though grade 12. The Grade 4 focus exhibit is *Miles of Tiles*.

MM2GO is designed to accommodate one class of up to 36 students at a time.

It is ideal to have only a small group of students at each exhibit while visiting *Math Midway 2 Go*. Break your class into six groups and have them rotate through the exhibits, with one group at each exhibit at a time. Before starting, make sure that students understand basic rules for interacting with the exhibits:

- Walk in the area surrounding the exhibits; don't run.
- Handle the exhibits gently.
- Do not hang or lean on the *Number Line Tightrope*.
- Handle *Ring of Fire* shapes gently.

Ideally, school support staff and/or parent volunteers will be available for the duration of the visit to *Math Midway 2 Go*. These adults can circulate throughout the exhibits, while the classroom teacher remains at the focus exhibit. At the five exhibits that are not the grade-level focus, students can explore and play.

Information about *Miles of Tiles*

About the exhibit:

Miles of Tiles is a magnet wall with various magnetic tile shapes–triangles, squares, hexagons, two types of rhombuses, and monkeys. As students play, they can explore many aspects of shape and pattern, observing how various tiles fit together. Note that *Miles of Tiles* is designed in particular to help students explore **tessellations**: patterns of tiles with no gaps and no overlaps, that could extend to cover the entire plane. Each of the shapes tessellates by itself, including the irregularly shaped monkeys, which most people find surprising. Some shapes will also tessellate in combination with other shapes (rhombus and square), whereas other combinations of shapes (square and hexagon) do not tessellate.

Why visit *Miles of Tiles*?

Elementary school students study shapes in depth, but they often are not exposed to the concept of tessellation. From tiles on a bathroom floor to patterns on fabric to the artwork of M.C. Escher, tessellations are all around us. *Miles of Tiles* provides a fun, playful extension to the study of shapes taking place in a 4^{th} grade classroom.

In addition to providing practice with the names of shapes, *Miles of Tiles* lends itself to the study of symmetry (both reflection symmetry and rotational symmetry), shape properties, and numerous advanced geometry concepts. Most teachers will be able to modify these activities to complement their classroom investigation of shapes.

The exhibit is both intuitive and thought–provoking. People of all ages are drawn to the exhibit. Questions that might be posed to students include the following: If someone else has already started a pattern, can you continue the tessellation to cover the entire board? Do all combinations of shapes tessellate? Without covering the board, can you predict whether your pattern will tessellate? How do you know?

Integrating MM2GO Into Your Unit Plans

Consider the following key questions, class topics, and elements of the Common Core State Standards when considering how to link the *Miles of Tiles* to the study of mathematics taking place in your classroom.

Key questions inspired by *Miles of Tiles*:

- How do shapes fit together?
- Can this pattern extend to cover a surface of any size, no matter how big? Why or why not?
- What is a tessellation and what properties does it have? Do tessellations have to repeat?

This lesson plan will be useful with the following classes:

- Classes studying shapes and their properties
- Classes studying tessellations for the first time

Relevant connections to the Common Core State Standards:

Learning Standards

4.G: Draw and identify lines and angles, and classify shapes by properties of their lines and angles. *Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.*

Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

Miles of Tiles Pre-Activity

Description

In this activity, students experiment with covering a sheet of paper using pattern blocks or other small shapes. Students experiment to see which shapes can be used to cover the paper with no gaps and no overlaps, and discover the concept of tessellation by the end of the lesson.

While this activity is designed for use before visiting *Miles of Tiles*, the activity can be enjoyed independently of a visit from the Museum of Mathematics' *Math Midway 2 Go*.

Materials

- One sheet of paper per student (any size or type of paper will work)
- Pattern blocks or other small shapes

  *Note: do **not** use the blue rhombus pattern blocks. MoMath's* Miles of Tiles *exhibit also has blue rhombuses, but the internal acute angle measure of the rhombuses from* Miles of Tiles *(72 degrees) is different from that of the rhombuses that are typically blue in a set of pattern blocks (60 degrees).*

  If you do not have access to pattern blocks, you can print the attached template on card stock. These paper pattern blocks were created using NCTM's Dynamic Paper applet (http://illuminations.nctm.org/ActivityDetail.aspx?ID=205), which allows teachers to create their own graphics.
- Optional: numerous small, common objects (paper clips, coins, etc)
- Optional: small mirrors

Key Terminology

- **Shapes,** including triangle, square, rectangle, trapezoid,
- rhombus, hexagon
- **Tessellation**

  Optional: **Symmetry**

Conducting the Activity

1. Seat students in groups, each group occupying its own table.

2. Give each group two pattern block shapes–one table might have (white) rhombuses and squares, another table might have triangles and trapezoids. Make sure that at least one group has two shapes that together do NOT tessellate, such as the hexagon and the square.

3. Pass out one piece of paper to each student.

Miles of Tiles Pre-Activity (Continued)

4. Challenge students to create a repeating pattern using only one shape that will completely cover the paper in front of them. They may not have enough supplies to cover the paper, but they should create a large enough design to demonstrate how they plan to fill the entire page.

5. Have students share with a partner, and ask–did you come up with the same pattern? Did you use the same shape?

6. Then, challenge the table to create a new design as a group, using both of the shapes. Can they create a pattern that uses both of the shapes, and which will be able to cover the entire sheet of paper? Some groups will be able to do so, but some may have difficulty. If groups find a solution quickly, ask them to save that design and try to design a second way to cover another sheet of paper.

7. Have students analyze their designs, looking for symmetry. Is there any point around which one can rotate the design less than a full turn so that it ends up looking the same as it did in its original position? If so, then the design has a rotational symmetry around that point. Is there any place where one could stand up a mirror so that the part of the design in the mirror would look the same as what's really there when you take the mirror away? If so, then the design has reflection symmetry along the line where the mirror was placed. Optionally, you can experiment with actual mirrors to find these lines of mirror symmetry.

8. Now, have a class discussion. Ask students for their observations. Keep a list of which shapes or combinations of shapes can cover the paper and which cannot.

9. At this point, introduce the terms **tessellate** and **tessellation** to students. Explain that when a shape or a combination of shapes covers an area of any size completely, with no gaps and no overlaps, the result is called a *tessellation*, and we say that those shapes *tessellate*.

   Stress to students that while they were only covering a single piece of paper, the design they made is a **tessellation** only if it can be extended to cover an area of any

Miles of Tiles Pre-Activity (Continued)

size whatsoever, no matter how large. Another way of saying this is that the design can cover an infinite flat plane. The idea of an infinite plane extending forever in all directions may be confusing; if you feel that it will not be helpful for your students, discuss the idea of covering larger and larger areas, no matter the size. The key point is that covering a piece of paper with a design does not by itself prove that a design is a tessellation; rather, the design must be able to continue forever in all directions to be a tessellation.

10. Show the attached images of different tessellations ranging from simple shapes (sidewalk bricks) to complex shapes (Islamic tile art). Then, show students the image of a bird sculpture, which is based on an M. C. Escher illustration. Explain that tessellations include designs made from complicated figures, like the birds Escher uses, which repeat and can cover a space of infinite size, again with no overlaps and no gaps. Tell students that they will make a complex drawing like this in a future activity.

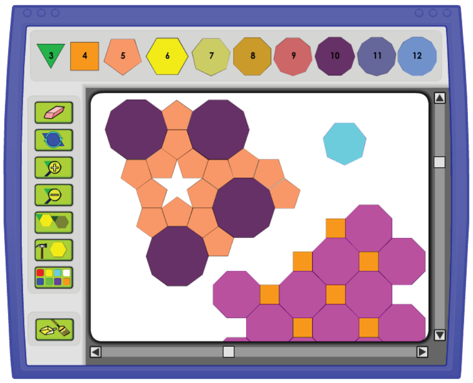
11. With the time remaining, allow students to use as many different pattern blocks as they choose to create a more complicated tessellation. Students can also choose to use the common objects to make a more unusual design–do paper clips tessellate? What shape would students need to fill in the gaps between them? Once students have a design they like, they can use any extra time to trace the design onto the paper, creating a work of tessellation art.

12. At the end of the class, explain that students will be using magnetic shapes to continue exploring tessellations during their visit to the exhibit *Miles of Tiles* at *Math Midway 2 Go*.

Miles of Tiles Pre-Activity (Continued)

Extensions

- Use a computer to access the Tessellation Creator on NCTM's *Illuminations* website: http://illuminations.nctm.org/ActivityDetail.aspx?ID=202
  Students can experiment with regular shapes, from triangles to dodecagons, discovering which combinations tessellate and which do not.



- Ask students to examine the world around them for tessellations. This can be done independently (as homework) or as a group exploration. Have students bring in photos or drawings of what tessellations they saw and where they saw them. Where do you find tessellations? Many examples may come from design–why would makers of products from floor tiles to wallpaper prefer to use tessellations?

Miles of Tiles Activity

Description

In this activity, students will explore *Miles of Tiles*, using the tiles to make patterns that do and do not tessellate.

Materials

- There are no specialized materials required for this activity other than the exhibit itself.

Key Terminology

- **Shapes,** including triangle, square, rectangle, trapezoid, rhombus, hexagon
- **Tessellation**
- Optional: **Symmetry**

Conducting the Activity

1. Allow students to explore the exhibit at their own pace first, using the magnets to make designs of their choice.

2. Once students have explored the exhibit for a few minutes, bring the group back together in a location where all of the students can see *Miles of Tiles*. Ask students what they have noticed so far.

3. Select a pattern that one student has started on *Miles of Tiles*. Stand so that all the students can see it. Ask students, "If you continued this pattern, could you cover the whole wall with it?" Have a discussion as a group about the pattern.

4. If a student uses the term **tessellation** that they learned in the pre-activity, ask them to explain that word. Make sure that all students understand that a tessellation is a pattern that can cover a surface of infinite size with no gaps and no overlaps.

5. Then, ask students which shapes they have found that tessellate by themselves, which shapes tessellate together with which other shapes, and which combinations of shapes do not tessellate. The monkey, for example, tessellates by itself, but not with any of the other shapes.

Miles of Tiles Activity (Continued)

6. If students are studying **symmetry** in the classroom, this might be a good opportunity to examine symmetry in their designs as well.

7. Allow students to use the remaining time to continue making designs with the *Miles of Tiles* magnets.

8. End by explaining to students that while there is no *Miles of Tiles* magnet wall in their classroom, they will be making their own special tessellating design, similar to the tessellating monkeys, in a future lesson.

Miles of Tiles Post-Activity

Description

In this activity, students will create their own tessellation pattern and create a tessellated piece of art.

While this activity is designed for use after visiting *Miles of Tiles*, the activity can be performed with students who have not had the opportunity to experience the Museum of Mathematics' *Math Midway 2 Go*.

Materials

- Blank paper, ideally in multiple colors
- Pre-cut squares, any size; 2 inches by 2 inches works well
- Pencils
- Scissors
- Tape
- Optional: rulers

Key Terminology

- **Shapes,** including triangle, square, rectangle, trapezoid, rhombus, hexagon
- **Tessellation**

Conducting the Activity

1. Ask students if they remember the tessellating monkey from the *Math Midway 2 Go*. What was so special about the monkey shape? Students may point out that the monkey is a complicated shape that tessellates. Explain to students that today they will be learning how to make their own complicated tessellating shapes starting from a basic shape that tessellates.

2. First, have students lay out nine squares each in a three-by-three grid, working alone or in groups, depending on how many squares you have available. Remind students that this is a portion of the familiar tessellation that squares can form.

3. Ask students–suppose you wanted to change one edge of this tessellation, so that instead of it being straight, it was curved? What would you have to do to the neighboring tile in the tessellation to prevent there from being a gap or overlap?

Miles of Tiles Post-Activity (Continued)

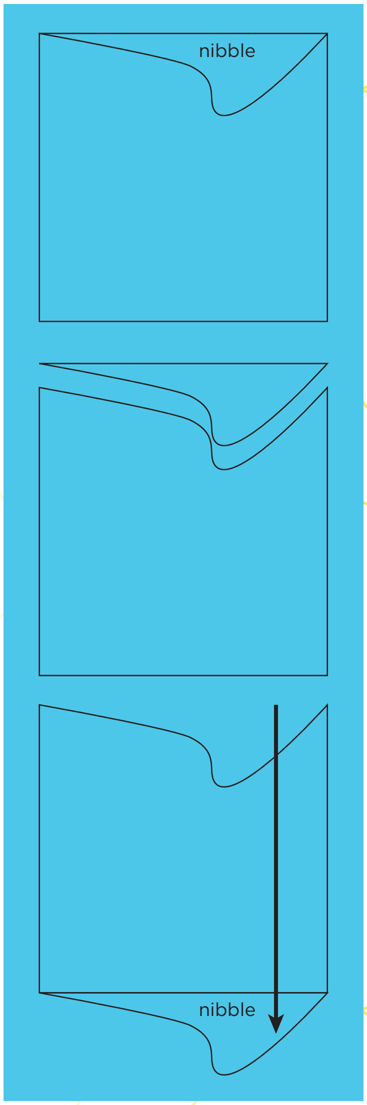
4. Have students visualize the answer to this question by drawing a curve that starts at one of the corners of one of the square tiles and ends at a neighboring corner. Ask them to imagine that the section of tile they outlined with their curve was added to the next tile over. How would the tile the student drew on have to be changed, in order to avoid an overlap? Help your students reach the answer that they would have to cut out that piece from the tile they drew on.

5. Now ask students, what if we wanted all of the tiles to be the same? Discuss this with students to reach the conclusion that it's necessary to make the same cut out of all of the tiles, and add each cut piece to the next tile over.

6. To make a new tessellation, every tile has to have a piece cut out, and every tile has to have a piece of that same shape attached to it on the opposite side of the cut.

7. That means we can actually make this change with a single tile. Ask your students if any of them can see how to do it.

8. Use the nibble diagram at right to help if needed. You should reach the conclusion that you can cut a shape out of one side, and attach it to the opposite side, to create a new shape that will still tessellate.



9. Now have your students actually do this physically. Have each student select one side of a precut square to change. Using a pencil, each student will draw what they intend to cut. When a student is happy with the design, s/he should use the scissors to cut out the design. Then the student should tape his or her cutout to the opposite side.

Miles of Tiles Post-Activity (Continued)

10. Optionally, you can repeat this step. If, for example, you have already modified the top and bottom sides of the square as shown in the nibble diagram, you can now modify the left and right sides.
Note that students do not have to cut the entire side from vertex to vertex. If they cut a smaller shape out of the side, students will have to use a ruler to measure how far from the vertex the cut began, and then mark the same length along the opposite side to be able to tape the cut part in the correct location along the opposite side.

11. When each student is happy with his or her modified shape, s/he should trace it onto a blank piece of paper, and cut out the resulting copy of the shape as a single piece of paper. Tell your students that as they are doing this, they should be careful to leave the rest of the paper as intact as possible, i.e., cut only along the traced lines.

12. Each student should now pick up his or her new cutout, and move it over and place it so it fits adjacent to the hole left by making the cutout. S/he should trace the cutout again in the new location, and then continue moving the shape so it lines up with the copies already traced, and tracing the shapes in the new locations. This process should be repeated until the whole page is covered. Finally, each student should cut up his or her page into the individual traced pieces. The result is a set of fascinating, homemade tiles that can cover as large an area as desired with no gaps and no overlaps – in other words, students have created their own tessellations.

Miles of Tiles Post-Activity (Continued)

Extensions

- Challenge students to find a way to use two colors of paper and their new shapes to create a two-color final artwork without coloring supplies. No copy of their tile shape should touch another copy of the same color, except at an isolated point or corner here and there.

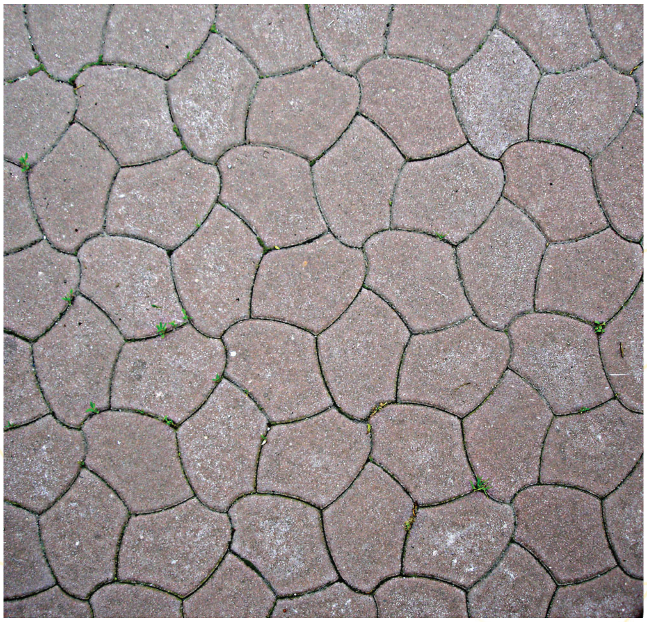
- Rather than tracing the shape over and over again, students can make their own stamps, using these instructions from PBS Teachers:
  http://www.pbs.org/teachers/connect/resources/2655/preview/
  Once a student has a stamp, s/he can use ink pads and a blank sheet of paper to stamp a diagram of the tessellation.

- In addition to shifting the cut paper from one side of the square to the opposite side, students can create a rotated tessellation. See the following unit plan for instructions, especially lesson 7 and the rotation diagram at the bottom of the page: http://mathcentral.uregina.ca/RR/database/RR.09.96/archamb1.html

- Have students begin with either equilateral triangles or regular hexagons – both of these shapes can tessellate. Challenge students to figure out how to modify either of these shapes to make a new, more complicated tessellating design, as they did with the squares. Modifying hexagons can be done either by the shift method in the main lesson above or by the rotation method from the previous extension. The rotation method works with triangles.

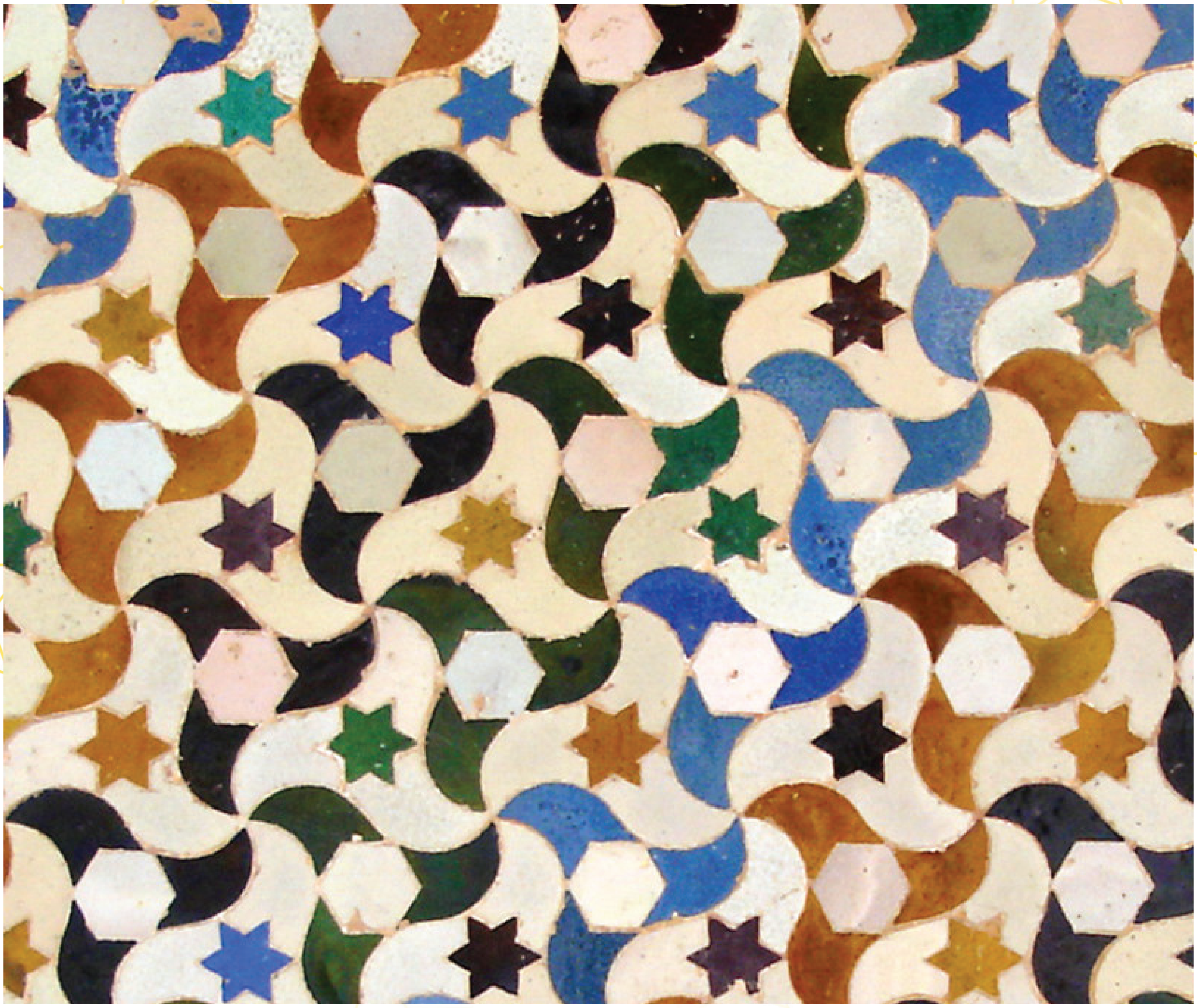
- Try a totally different method of creating a shape that will tessellate, using the "paper cut method": http://www.tessellations.org/methods-diy-papercut.shtml

- Explore the work of Makoto Nakamura, the Japanese tessellation artist who designed the tessellating monkey from the Miles of Tiles exhibit:
  http://www.k4.dion.ne.jp/~mnaka/home.index.html

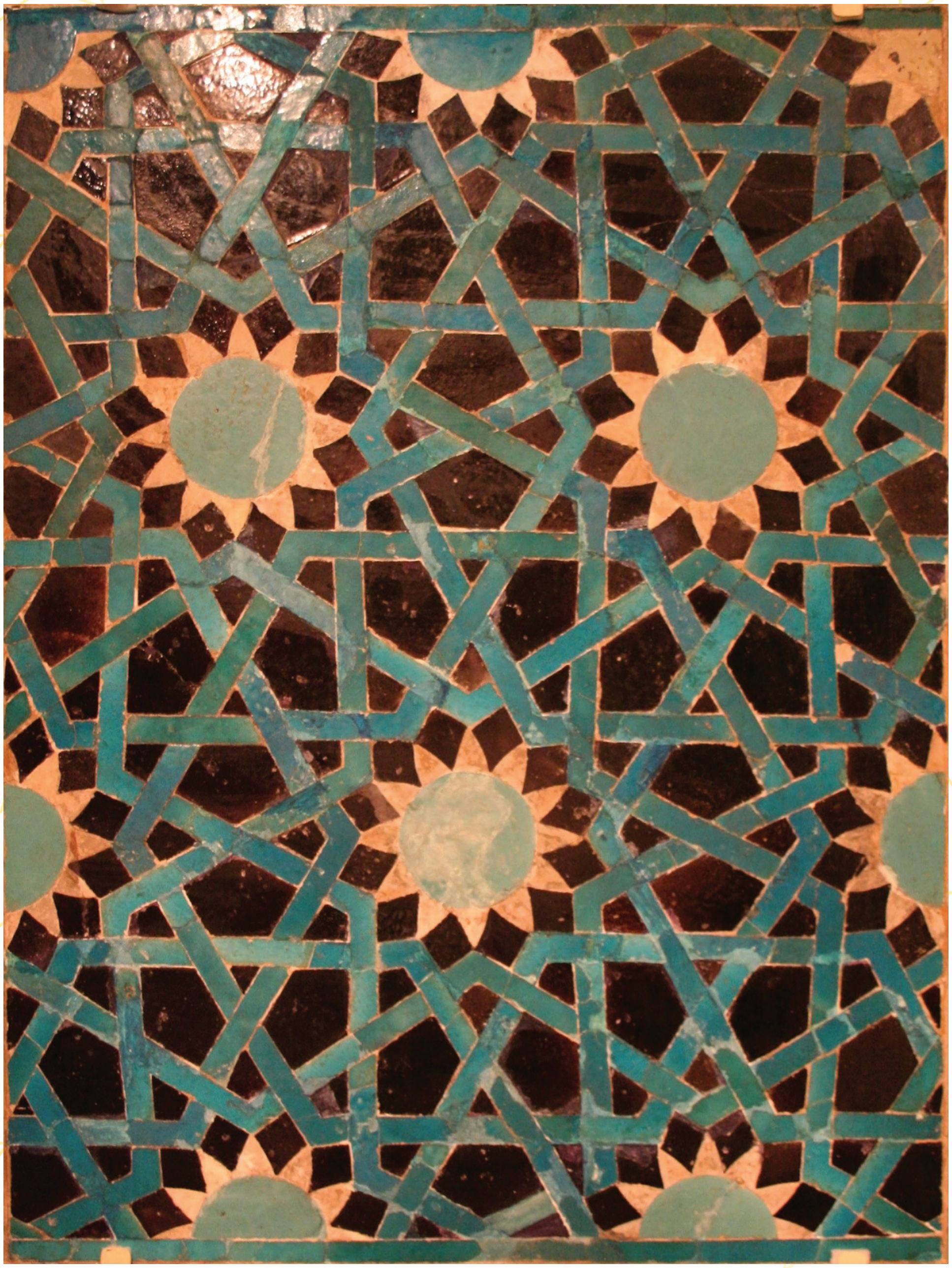


By Dmharvey [Public domain], via Wikimedia Commons

By uploader [Creative Commons], via Wikimedia Commons



By Dmharvey [Creative Commons], via Wikimedia Commons



By Ophelia2 [Public domain], via Wikimedia Commons

By Bouwe Brouwer [Creative Commons], via Wikimedia Commons

Paper Pattern Blocks

Name ______________________