MATH MIDWAY 2GO

NUMBER LINE TIGHTROPE

Lesson Plan

GRADE
6

Brought to you by MoMath

A Letter from the PSEG Foundation

My fascination with energy started at a young age.

The Arab oil embargo of the 1970's sent gasoline prices through the roof and made clear how closely tied our country's foreign policy is to oil interests. I began wondering whether we could produce energy in ways that didn't involve oil, and I wanted to be part of the quest to find the answer.

That passion led me to pursue years of study in the fields of physics and engineering. Graduate degrees in those areas allowed me to take on a variety of fascinating assignments in my career. I served as a research scientist at the Princeton Plasma Physics Lab, a Congressional Science Fellow in the office of U.S. Senator Bill Bradley, and a science, energy, and technology policy advisor to Governor Tom Kean before coming to PSEG where I work every day to create and deliver power responsibly.

This curriculum, developed by the Museum of Mathematics and funded by PSEG, is intended to help young people develop an interest in math and the technical fields–to spark curiosity, stimulate inquiry, and help students down a path of discovery that leads to fulfilling careers.

As issues such as climate change, energy independence, and national security demand increasingly comprehensive and technical solutions, the need for people with knowledge in science, technology, engineering, and math–areas known as the STEM subjects–will continue to grow.

At PSEG, we understand that our country's future depends on developing the insights, creativity, and dynamism of the next generation of innovators. This curriculum is one of many investments we've made in an effort to help young people discover their talents and develop a thirst for knowledge.

A math- and science-savvy workforce will lead the way to innovative technological discovery, a strengthened economy, and thriving new industries. And it is an important part of building a talent pipeline for the energy industry and our country as a whole.

Ralph Izzo
Chairman, CEO and President, PSEG

MoMath is pleased to acknowledge the support of the
Alfred P. Sloan Foundation in the creation of *Math Midway 2 Go*, and the support
of the PSEG Foundation in the creation of the accompanying curriculum.

Alfred P. Sloan Foundation

PSEG Foundation

Table of Contents

General Instructions for *Math Midway 2 Go*

Math Midway 2 Go (MM2GO) consists of six interactive mathematics exhibits that can travel to schools and other venues. Hands-on activities captivate and engage students, highlighting the wonder of mathematics. These exhibits were designed for use with individuals of all ages, and the mathematical topics they address range from topics in the elementary classroom to college-level mathematics. Students of all ages will benefit from open exploration of the exhibits. At the same time, the exhibits also tie into specific curricular topics for kindergarten through grade 12.

These lesson plans are provided by MoMath to support teachers like you. To help you and your students make the most of your time at *Math Midway 2 Go*, a focus exhibit has been selected for each grade from kindergarten though grade 12. The Grade 6 focus exhibit is *Number Line Tightrope*.

MM2GO is designed to accommodate one class of up to 36 students at a time.

It is ideal to have only a small group of students at each exhibit while visiting *Math Midway 2 Go*. Break your class into six groups and have them rotate through the exhibits, with one group at each exhibit at a time. Before starting, make sure that students understand basic rules for interacting with the exhibits:

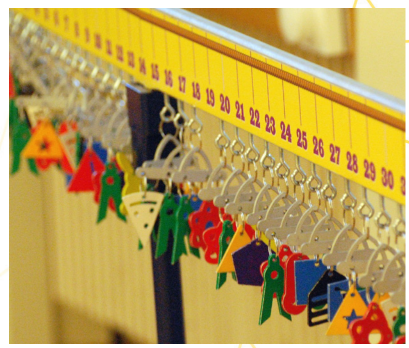
- Walk in the area surrounding the exhibits; don't run.
- Handle the exhibits gently.
- Do not hang or lean on the *Number Line Tightrope*.
- Handle *Ring of Fire* shapes gently.

Ideally, school support staff and/or parent volunteers will be available for the duration of the visit to *Math Midway 2 Go*. These adults can circulate throughout the exhibits, while the classroom teacher remains at the focus exhibit. At the five exhibits that are not the grade-level focus, students can explore and play.

Information about the *Number Line Tightrope*

About the exhibit:

The *Number Line Tightrope* features the numbers from -10 to 100, many of which are decorated with colorful hanging iconic shapes, arranged on a long horizontal beam. Each icon represents a different number family. As students explore, they link their observations with their prior knowledge of numbers and their properties. For example, students might try to figure out why certain numbers have a square hanging from them, while others do not. There are fourteen different number families to explore. Some are quite challenging, but even the youngest students can understand the square numbers, or learn why we call some numbers “triangular.”



Why visit the *Number Line Tightrope*?

Students in both primary and secondary grades are learning about number families. They already know primes and composites, and they are being introduced to squares and cubes. The *Number Line Tightrope* includes all of these, as well as factorials, triangular numbers, pentagonal numbers, tetrahedral numbers, highly composite numbers, pizza numbers, cake numbers, perfect numbers, constructible polygon numbers, powers of two, and Fibonacci numbers, along with some guest irrational numbers.

Each of the number families has a different symbol. During their visit to the *Number Line Tightrope*, students use their observation skills to notice patterns. The red atom is found at the numbers 2, 3, 5, 7, 11, and so on. What could it represent?

While students may not have the content knowledge to discover the meaning of all the symbols, curiosity and close observation together with support from the pre-activity (designed to prepare students) and post-activity (designed to expand on their hands-on investigations) should allow students to decipher a handful of the symbols.

Integrating MM2GO Into Your Unit Plans

Consider the following key questions, class topics, and elements of the Common Core State Standards when considering how to link the *Number Line Tightrope* to the study of mathematics taking place in your classroom.

Key questions inspired by the *Number Line Tightrope*:

- What makes a family of numbers?
- Which number families come from patterns and which do not?
- What kind of rule makes up a given family of numbers?
- Can you use the answers to the previous two questions to determine the next number in this number family?
- Why do some number families have positive and negative members while others have only positive members?

This lesson plan will be useful with the following classes:

- Middle school classes learning about integers and/or exponents
- Classes studying any of the following: primes, squares, cubes, the Fibonacci sequence, triangular numbers, perfect numbers, pentagonal numbers, tetrahedral numbers, or powers of two

Relevant connections to the Common Core State Standards:

Learning Standards

6.NS: Apply and extend previous understandings of numbers to the system of rational numbers. *Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.*

Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Attend to precision.
- Look for and make use of structure.
- Look for an express regularity in repeated reasoning.

Number Line Tightrope Pre-Activity

Description

In this activity, students will investigate number families that are already familiar to them, grouping numbers in a variety of ways.

While this activity is designed for use before visiting the *Number Line Tightrope*, the activity can be enjoyed independently of a visit from the Museum of Mathematics'

Materials

- Chart paper
- Sticky notes

Key Terminology

- **Number groups** or **number families**
- Categories such as **even, odd, prime, composite, positive, negative, counting number, whole number, integer, square, cube**

Students may well find other number families, and their use of mathematical terminology should be encouraged.

Conducting the Activity

1. Give each student five sticky notes and have them write down any five numbers, one number on each note.

2. Ask each student to pick a criterion that could describe some or all of the numbers, and split the notes into those that fit the criteria and those that do not. Walk around and investigate their groupings. Have students compare with a partner at their table—how did each student split the numbers?

   You can ask students follow-up questions. If students have split into groups like even and not even, you could ask, "What is another way of expressing not even?" With the groups prime and composite, you might ask, "Is there any number you can think of that would not fall into either category?"

Number Line Tightrope Pre-Activity (Continued)

3. Once students have shared with a partner, ask them to re-group their numbers based on a totally new criterion that neither they nor their partner used the first time. What other number labels can students find?

4. Have students share this new grouping with a partner.

5. After sharing, ask the whole class: what groupings have you used so far? Record a complete list of class groupings on a piece of chart paper or the blackboard.

6. Next, have students work in groups of four to six to take all their numbers and group them together. Students will have to agree on a grouping criterion, splitting the numbers into two categories. Once they have picked their criterion, have them record their split on a piece of chart paper and stick all of their individual numbers to the group chart paper.

7. When all groups are ready, have each group share their work with the whole class. Did groups pick similar or different ways to group the numbers? What other ways could numbers be grouped that are not yet represented? If two groups have the same criterion (for example, numbers split into positive and negative), do they have all the same numbers listed? You can point out here that many number families have infinitely many members.

8. If time permits, have each student pick one number. The student can then use as many sticky-notes as needed to paste that number on all the pieces of chart paper on which it belongs. For example, if seven were selected, it would be placed in the odd group, the prime group, the positive group, and whatever other appropriate number families have been listed by the class.

9. End the class by explaining that students will be exploring new number families at the *Number Line Tightrope* during their visit to *Math Midway 2 Go*.

Number Line Tightrope Pre-Activity (Continued)

Extension One: Venn Diagrams

Students can practice making mathematical Venn Diagrams with their number groups. Students pick two terms, such as prime and odd, square and even, or any other two terms that can be used to describe numbers. Then, students can create a Venn Diagram showing which numbers belong to only one label or the other, which belong to both (if any), and which belong to neither (again, if any.)

Extension Two: Art with Number Families

Students can be asked such questions as, “What image would you make to represent each number family? What picture could you draw to represent prime, or odd, or square?” Then, each student can pick a label and draw an image to represent it, explaining why the picture references the label.

Number Line Tightrope Activity

Description

In this activity, students will explore the *Number Line Tightrope*, using their observation and reasoning skills to determine the meaning of the symbols they see.

Materials

- Attached *Number Line Tightrope Observation Sheet*, one copy per student
- Attached *Guide to the Number Line*, a few copies for reference
- Pencils
- Optional: clipboards

Key Terminology

- **Number groups** or **number families**
- Categories such as **even, odd, prime, composite, positive, negative, counting number, whole number, integer, square, cube**

Students may well find other number families, and their use of mathematical terminology should be encouraged.

Conducting the Activity

1. Allow students to examine the exhibit at their own pace.

2. Once students have examined the exhibits for a few minutes, bring the group back together. Ask students what they have noticed so far.

3. After students have shared their observations, hand out the *Number Line Tightrope Observation Sheet*. Explain to students that they should use the sheet to record the numbers that go with a particular symbol and their theories about that symbol. Students can work individually, in pairs, or in small groups to perform this investigation.

4. While students explore, make yourself available with copies of the *Guide to the Number Line*. If students have theories about a given symbol, engage them in conversations. Ask students: what do you think the symbol means? Why do you think that? What questions do you have?

Number Line Tightrope Activity (Continued)

5. You can either give students a hint to help them if they are confused, or you can hand them the *Guide to the Number Line* and allow them to check their ideas against the guide.

6. At the end of the exploration time, after all groups have rotated through the *Number Line Tightrope*, have a discussion with the entire class. Have students sit down near the *Number Line Tightrope* and talk about their experiences exploring the exhibit. For which symbols were students able to deduce the meaning? Which symbols stumped the class? What new ideas did the class learn from observing the *Number Line Tightrope*?

   Note that some symbols touch on topics that require more advanced mathematical knowledge.

7. End by explaining to students that while visiting the *Number Line Tightrope* was a one-time, special experience, they will be making a permanent, symbol-filled number line back in the classroom.

Number Line Tightrope Post-Activity One

Description

In these activities, students will review the number families introduced by the *Number Line Tightrope* and have the opportunity to learn about a few number families that may not have been familiar to them.

This post-activity is designed for use after visiting the *Number Line Tightrope*. Note that Post-Activity Two may be enjoyed by students who have not had the opportunity to experience the Museum of Mathematics' *Math Midway 2 Go*.

Materials

- Attached *Guide to the Number Line*, a few copies for reference
- Attached *Number Line Tightrope Number Family Explanations and Examples*, one copy for the teacher

Key Terminology

- **Number groups** or **number families**
- Terms from the *Number Line Tightrope*: **cake number, counting number, cube, factor, factorial, Fibonacci number, highly composite number, integer, pizza number, power of two, prime, square, whole number, zero, constructible polygon number, pentagonal number, perfect number, triangular number, tetrahedral number**

Conducting the Activity

1. Ask students: what did you discover during your exploration of the *Number Line Tightrope*? What do you remember?
2. Have students share with a partner the most interesting thing they learned or discovered while interacting with the *Number Line Tightrope*.
3. Then, discuss with the whole class. Have students report what they shared with their partner while the teacher takes notes.
4. Ask: which symbols stumped you? Where did you see these symbols? What theories do you have about them?

Number Line Tightrope Post-Activity One (Continued)

5. At this point, explain to students that you will not have time to explain all the concepts on the *Number Line Tightrope*, but that they are welcome to do research on their own. Take the time to teach a few of the concepts students do not yet know.
   - If your students are not already familiar with squares and cubes, this is one opportunity to touch on those topics.
   - Powers of two and highly composite numbers are concepts with which students may not be familiar, and which would be simple to explain to 6th graders.
   - Fascinating number families that are less familiar to students include pizza numbers, Fibonacci numbers, and factorials. Use the attached *Number Line Tightrope Number Family Explanations and Examples* to guide students' understanding of the number families.

6. When teaching an unfamiliar number family, have students start with their notes from the exhibit visit—when did they see the pizza symbol? Which numbers are pizza numbers? Who has a theory as to how pizza numbers are determined? Work with students to figure out the symbol, supplying leading questions, hints, and new information as needed.

Number Line Tightrope Post-Activity Two

Description

In these activities, students will review the number families introduced by the *Number Line Tightrope,* and have the opportunity to learn about a few number families that may not have been familiar to them.

This post-activity may be enjoyed by students whether or not they have had the opportunity to experience the Museum of Mathematics' *Math Midway 2 Go*.

Materials

- Blank paper, ideally multiple colors
- Coloring supplies—crayons, markers, colored pencils
- Scissors
- Tape

Key Terminology

- **Number groups** or **number families**
- Terms from the *Number Line Tightrope*: **cake number, counting number, cube, factor, factorial, Fibonacci number, highly composite number, integer, pizza number, power of two, prime, square, whole number, zero, constructible polygon number, pentagonal number, perfect number, triangular number, tetrahedral number**

Conducting the Activity

1. Explain to students that this activity will turn the classroom's existing number line into a colorful display of the number families the class is investigating.

2. Ask students: what number families should be on our number line? They can be categories found on the *Number Line Tightrope* or any other family of numbers that students suggest.

3. Make a list of the number families students suggest. Then, split your class into groups to cover all the listed number families. For example, if students want to add seven number families to your number line, split students into seven groups.

Number Line Tightrope Post-Activity Two (Continued)

4. Each group will be responsible for one number family. They will decide together what symbol will be used for that number family, and which numbers between negative ten and one hundred correspond to their number family (if your classroom number line has a different range, adjust accordingly.) Once they have decided upon both the symbol and the numbers needed, students are responsible for drawing sufficiently many copies of the symbol and attaching them to hang below the classroom number line at all appropriate locations.

5. Students should also make a key to be placed somewhere near the number line, explaining what their symbol means.

6. Students who finish early may be able to help other groups. For example, if a group has selected "even" as their number family, they will need a large number of pictures. Students from other groups may be able to help with the drawing.

7. Once all groups are finished, have each group share its symbol, number family, and the locations of those symbols along the number line.

You now have a classroom number line, just like Math Midway 2 Go!

Number Line Tightrope Post-Activity Two (Continued)

Extension One: Prime Numbers

Explore the Sieve of Eratosthenes (using page 18) with your students to find all of the prime numbers from 1 to 120. Here's how it works:

By definition, 1 is not prime, so cross it out. To find all the primes, repeatedly follow these two simple steps: first, find the smallest number that is not crossed out and circle it, because it is the next prime number. Second, cross out all of the multiples of the number you just circled, since they can't possibly be prime. Let's see how it goes. After crossing out the 1, the smallest number not crossed out is 2. That means that 2 is prime, so circle it. Then cross out 4, 6, 8, and so on—all the multiples of two, or in other words, all of the even numbers. Once that's done, the smallest number not crossed out is 3, so that's the next prime. Circle it, and cross out 6, 9, 12, and so on—all of the multiples of three. (Actually, you will find that half of the multiples of three were already crossed out in the previous round, because they are even.) At this point, the smallest number remaining is 5, so that gets a circle indicating it's prime, and now 10, 15, 20, and so on get crossed out—all of the multiples of 5. Continue alternating circling the smallest remaining number and crossing out all of its multiples, and you will end up circling all of the prime numbers less than 120. A key showing the correct final layout is included on page 19.

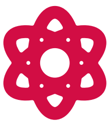
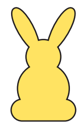
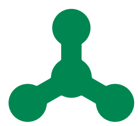
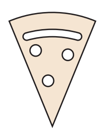
Extension Two: Fractions

There were no fractions on the *Number Line Tightrope*. Choose some you consider appropriate, and ask students to place these fractions on the number line.

Extension Three: Venn Diagram

Repeat the Venn diagram activity from the pre-lesson with the new number families that students learned through exploring the *Number Line Tightrope*.

Number Line Tightrope Observation Sheet

	
	
	
Your Choice	Your Choice

Sieve of Eratosthenes

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

Sieve of Eratosthenes Answer Key

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~
101	~~102~~	103	~~104~~	~~105~~	~~106~~	107	~~108~~	109	~~110~~
~~111~~	~~112~~	113	~~114~~	~~115~~	~~116~~	~~117~~	~~118~~	~~119~~	~~120~~

Number Line Tightrope Number Family Explanations and Examples

Fibonacci Sequence

Leonardo of Pisa, also called Fibonacci, started a list of numbers with 0 and 1. To find the next number, he added the last number to the number before: 1+0=1. The list was then 0, 1, 1. He repeated this process to find the fourth number in the list: 1+1=2. Then the list was 0, 1, 1, 2. Continuing this process generates the Fibonacci sequence.

Fibonacci numbers on the *Number Line Tightrope*: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89
Beyond the Century: What are the next two numbers in the Fibonacci Sequence? 144, 233

Square Numbers

Using square blocks, make squares that start small and then get slightly bigger. The smallest square uses only 1 square tile. The next larger square you can make uses 4 square tiles. The following square uses 9 tiles. Students working in groups can come up with the family of square numbers even before they learn how that relates to multiplication and/or exponents.

0 is also a square number, but this fact may be omitted when working with young students.

Square numbers on the *Number Line Tightrope*: 0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100
Beyond the Century: What are the next two square numbers? 121, 144

Cubes

Students will need prior knowledge either of three-dimensional cubes or of exponents. Start by having students identify the square numbers on the *Number Line Tightrope*. Then point out that the square symbol and this other blue symbol look similar (make sure you don't say "cube" yet–it gives away the answer too soon). Ask students to figure out where this symbol is found and why it might look like the square. At this point, a student who is familiar with either drawing cubes or exponential notation will figure out that the symbol means cubes.

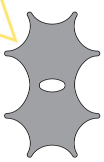
Number Line Tightrope Number Family Explanations and Examples (Continued)

Have students prove it–pick a positive cube, like 27, and ask: what number cubed gives 27? The answer is 3 because 3 * 3 * 3 or 3^3 is 27. Have students find 1^3, 2^3, and 4^3. Then, ask students what 5^3 would be. The answer is 125, which is beyond this number line.

Note that 0 is also a cubed number–help students identify that 0 is the value of 0^3. If students are comfortable with negatives, ask students: are all the cubes positive? The answer is no, since -1 and -8 are cubes. Help students figure out why this is the case: -1 is the cube of -1 and -8 is the cube of -2.

Cubes on the *Number Line Tightrope*: -8, -1, 0, 1, 8, 27, 64
Beyond the Century: What are the next two cubes? 125, 216

Powers of Two



Ask students to find this symbol anywhere on the number line. Then, have them carefully look for the preceding and following instances of the symbol. So, if they found 16, they would look to find 8 before and 32 after. Ask students: what is the relationship between these numbers? If students haven't figured it out, have them keep looking for preceding and following instances of the symbol. At some point, a student will figure it out–the following number is always double and the preceding one is half. Then ask students to find all the instances of this symbol on the *Number Line Tightrope*–where does the pattern start? It starts at 1 and doubles each time to get to 64. The following double, 128, is off the number line. This family of numbers is called Powers of Two.

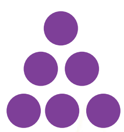
If students are comfortable with exponents, you can link the name of the family to exponential notation–each of these numbers can be written as two to the power of something, which is to say 2^x. Practice by taking each number with the Powers of Two symbol and figure out what the value of x is for each.

Powers of Two on the *Number Line Tightrope*: 1, 2, 4, 8, 16, 32, 64
Beyond the Century: What are the next two Powers of Two? 128, 256

Number Line Tightrope Number Family Explanations and Examples (Continued)

Triangular Numbers

Ask students to start at 0 and then find the following triangular number: 1. Ask them how the number grew. (It went up by 1.) Then find the following triangular number: 3. Ask students how the number grew this time. (It went up by 2.) Repeat this investigation until students see the pattern–each time, you add the following whole number (+1, +2, +3, +4, etc.) So, why are these numbers called triangular numbers? Use counting chips or coins to build triangles. Start with one coin–this could have an equilateral triangle built around it. Then, add a row of two coins below–this now looks like a larger equilateral triangle. For the next row, ask students: how many coins will we add? Have a student demonstrate adding three coins to make a triangle using six coins in total:

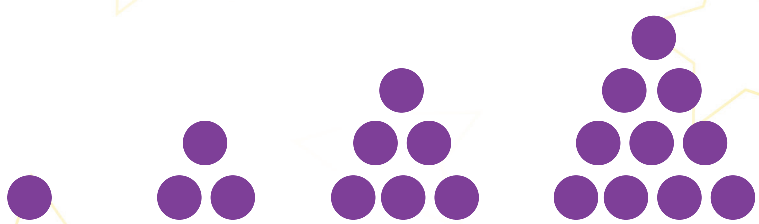


6

Have students build ever-larger equilateral triangles side by side and link these physical triangles to the yellow triangle symbol on the number line. Triangular numbers are the numbers of coins or tiles that can be used to make physical equilateral triangles.

0 is also a triangular number, but it is okay to omit it when working with young students.

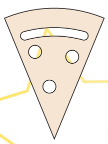
Triangular numbers on the *Number Line Tightrope*: 0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91
Beyond the Century: What are the next two triangular numbers? 105, 120



1 3 6 10

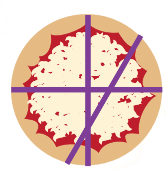
Number Line Tightrope Number Family Explanations and Examples (Continued)

Pizza Numbers



This number family works best if you have a drawing surface and can draw a diagram as you go.

Tell students that they work in a very silly pizza shop. They start with an entire pizza and can cut only straight lines. If they make zero cuts, how many slices will they have? They will have 1 very large slice. Now, they get to make one cut–how many slices now? There are 2. Cut one more time (two cuts) and there are 4 slices. Explain that here is where it gets crazy–how many slices can students make with the third cut? Students will typically say 6, or perhaps 8 (but that requires making two more cuts). Have students draw how they would get to 6–they will draw a third line going through the intersection of the first two lines. Then, remind students that this is a silly pizza shop–unlike a standard pizza shop, students do NOT have to cut through the intersection, or through the center. Ask students: could you go from 4 slices to more than 6 slices with the third cut? Eventually, a student will show with their finger or a writing implement that if you cut through three regions, you can get to 7 slices:



This list (1, 2, 4, 7) is the beginning of the pizza numbers. Ask students–is there a pattern? Yes: like the triangular numbers, the pizza numbers grow each time according to a pattern–each time you add the following whole number (+1, +2, +3, +4, etc.). Use this pattern to predict the next pizza numbers and then verify those guesses by looking at the *Number Line Tightrope*. Of course, these are slices of unequal size, but it is possible to get 92 slices with 13 cuts!

Pizza numbers on the *Number Line Tightrope*: 1, 2, 4, 7, 11, 16, 22, 29, 37, 46, 56, 67, 79, 92
Beyond the Century: What are the next two pizza numbers? 106, 121

Number Line Tightrope Number Family Explanations and Examples (Continued)

Prime Numbers

Ask students to figure out what is true about all the numbers that have this red atom symbol. Students are often quick to notice that these numbers are all odd. However, point out an odd number missing the red atom, such as 51 or 9. Then, ask students if there are any even numbers with the red atom symbol. Help them find the 2 if they have not found it themselves. At this point, students who are already familiar with primes tend to figure out the family. If they do not, use the Sieve of Eratosthenes activity in the 6th grade *Number Line Tightrope* lesson plan to help students find all the primes through 100.

Prime numbers on the *Number Line Tightrope*: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
Beyond the Century: What are the next two prime numbers? 101, 103

Highly Composite Numbers

Understanding this number family requires prior knowledge of factors, primes, and composites. Make sure students have discovered already which symbol represents primes. Ask them what the opposite of prime is–composite. Ask students: does the *Number Line Tightrope* have a symbol for composites? No–for example, 18 is certainly composite, yet it has no symbol hanging down. Explain to students that while there is no composite symbol, there is something new and cool to find on the *Number Line Tightrope*. Have students walk to 1 on the number line and ask: is 1 prime or composite? It is neither. Ask the same question of 2: is it prime or composite? It is prime. Ask students–what is the first composite number? The correct answer is 4. Ask students to list all the factors of 4: 1, 2, and 4. The number 4 has three factors. Ask students: what is the first number that will have more than three factors? Check the students' suggestions–the correct answer is 6, whose factors are 1, 2, 3, and 6. The number 6 has four factors. What number will be the first to have more than four factors? The answer is 12: 1, 2, 3, 4, 6, and 12. The number 12 has six factors. Now that students are getting the pattern, explain to students that this green symbol found at 4, 6, and 12 represents the family of highly composite numbers–positive

Number Line Tightrope Number Family Explanations and Examples (Continued)

composite numbers that have more factors than any prior positive number on the number line. Allow students to find the other highly composite numbers and figure out how many factors each one has.

Highly composite numbers on the *Number Line Tightrope*: 4, 6, 12, 24, 36, 48, 60
Beyond the Century: What are the next two highly composite numbers? 120, 180

Perfect Numbers

This one requires knowledge of factors. Explain that 6 and 28 are special numbers called perfect numbers. Ask students to name all the factors of 6: 1, 2, 3, and 6 itself. Now, ask students to add all the factors other than the number itself (6). They will find that 1+2+3 equals 6. Try the calculation now with a non-perfect number, like 10. The factors of 10 (other than 10 itself) are 1, 2, and 5. Adding them gives 8, which is smaller than 10. Now, try with 12. The factors of 12 (other than 12 itself) are 1, 2, 3, 4, and 6. Adding them gives 16, which is bigger than 12. There are, in fact, very few numbers whose factors (other the number itself) add up to the number. Greek mathematicians discovered the first of these numbers and called them "perfect" because they were so special. Have students check 28 to ensure it is indeed perfect–start by listing the factors of 28 other than 28 itself: 1, 2, 4, 7, and 14. Add them–the sum is 28. These are the only perfect numbers on the *Number Line Tightrope*; the next perfect number is actually 496.

If students happen to notice that this symbol looks like the symbol for triangular numbers, that is intentional–all perfect numbers are also triangular numbers.

Perfect numbers on the *Number Line Tightrope*: 6, 28
Beyond the Century: What are the next two perfect numbers? 496, 8128

Number Line Tightrope Number Family Explanations and Examples (Continued)

Factorial Numbers

Ask students what an exclamation point means when they are writing. After students relate their thoughts, explain that in math, the exclamation point has a special meaning–it is called a factorial. 3! means to multiply 3 * 2 * 1–what does that equal? The answer, 6, is a factorial number. Ask students to predict–what would 4! be? Have students try to generalize: if 3! starts with 3 and then multiplies it by the smaller whole numbers down to 1, see if students can generate the list 4 * 3 * 2 * 1 for 4!. Then, evaluate: 4! is equal to 24. So, 24 is also a factorial number. Then, have students figure out 1!, 2!, and 5!. Make sure they notice that 1!, 2!, 3!, and 4! are all on the *Number Line Tightrope* while 5!, which is 120, is too high to show up here.

Factorial numbers on the *Number Line Tightrope*: 1, 2, 6, 24
Beyond the Century: What are the next two factorial numbers? 120, 720

Additional Number Families

Cake numbers are a three-dimensional version of pizza numbers.
Cake numbers on the *Number Line Tightrope*: 1, 2, 4, 8, 15, 26, 42, 64, 93
Beyond the Century: What are the next two cake numbers? 130, 176

Constructible polygon numbers are the numbers of edges of the regular polygons that can be constructed using only a compass and a straightedge.
Constructible polygon numbers on the *Number Line Tightrope*: 3, 4, 5, 6, 8, 10, 12, 15, 16, 17, 20, 24, 30, 32, 34, 40, 48, 51, 60, 64, 68, 80, 85, 96
Beyond the Century: What are the next two constructible polygon numbers? 102, 120

Number Line Tightrope Number Family Explanations and Examples (Continued)

Pentagonal numbers are the pentagonal extension of the triangular and square numbers.
Pentagonal numbers on the *Number Line Tightrope*: 1, 5, 12, 22, 35, 51, 70, 92
Beyond the Century: What are the next two pentagonal numbers? 117, 145

Tetrahedral numbers extend triangular numbers into three dimensions, making tetrahedra rather than triangles. Tetrahedral numbers are to triangular numbers as cubes are to square numbers.
Tetrahedral numbers on the *Number Line Tightrope*: 1, 4, 10, 20, 35, 56, 84
Beyond the Century: What are the next two tetrahedral numbers? 120, 165

Guide to the Number Line

Come visit a world of Platonic perfection!

Like other exotic realms, the Number Line has its own geography, its own special attractions, and its own key sights to see. Some of these highlights are steeped in history, while others are more recent discoveries. The Number Line will help you orient yourself in the world of numbers.

Primes

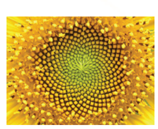
Don't miss the prime numbers! Since arithmetic was invented, people have tried to understand how multiplication connects different integers. For example, 15 equals 3x5, so 3 and 5 are called "factors" of 15. You can divide most numbers into factors in a similar way. But some numbers, called the "primes," cannot be broken down further. Ancient Greek philosophers named the smallest bits of matter "atoms," which means "indivisible." Primes are the atoms of the number world—they are the indivisible building blocks of all of the integers. As atoms combine to make molecules, so primes can be multiplied together to form all the other integers.

Squares

The square numbers are another must-see. This fascinating family of numbers comes from multiplying each integer by itself. Imagine, for example, a square made up of four rows of four dots each. That diagram would contain 4x4=16 dots altogether, so 16 shows up in the pattern of square numbers.

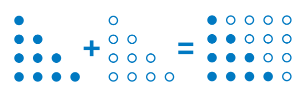
Fibonacci Sequence

In an arithmetic textbook in the year 1202, Leonardo of Pisa (also known as Fibonacci) published an innocuous word problem about raising rabbits and touched off centuries of mathematical discovery. He asked how many pairs of rabbits you would have after a year if you started with one pair, and if every pair at least two months old produced a new pair of rabbits each month. Answering this problem leads to the sequence of numbers shown here, in which each number is the sum of the previous two. Although the sequence doesn't actually tell you much about real rabbits, it does come up over and over again in nature—for example, the number of spirals of seeds in a sunflower is almost always a Fibonacci number.



Triangular Numbers

The story is told about the great mathematician Gauss that in 1784 as a schoolboy, his class was assigned to add up every number from 1 to 100. Much to the teacher's amazement, Gauss returned with the correct answer—5050—in less than a minute! This sum can be thought of as the number of dots in a triangular array, and two triangles can go together to form a rectangle, easily counted using multiplication. It seems that as a schoolboy, Gauss discovered the formula n(n+1)/2 for the nth triangular number. Can you see the pattern in the triangular numbers shown here?



Perfect Numbers

At least since the times of the Greek philosopher Pythagoras, visitors to the number line have been comparing numbers to their factors (see the Primes.) One way to compare is to add up all of the smaller factors of a number. For most numbers, like 21, the sum you get (1+3+7=11) will be less than the starting number. For many other numbers, like 12, the sum (1+2+3+4+6=16) will be more than the original number. And for just a handful of numbers, like 28, the sum comes out exactly equal to the original number: 1+2+4+7+14=28. Given their rarity, Greek mathematicians called these numbers "perfect." Every even perfect number must be a triangular number, so we've indicated them with a star on their triangle. Nobody knows whether there are any odd perfect numbers—maybe you can solve that mystery someday!

Guide to the Number Line (Continued)

Factorials

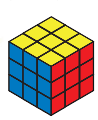
The factorials are the multiplication version of the triangular numbers: instead of adding up the first several numbers, you multiply them all together. For example, the 5th factorial number, written 5!, is 1x2x3x4x5. All that multiplication makes the factorial numbers get big fast, so you won't see too many on this portion of the Number Line. Factorials come up all the time in calculating the probabilities of things. For example, the chance that you are dealt a spades royal flush in poker is 1 in 52!/5!47!, or 1 in 2,598,960.

Powers Of Two

You'll definitely want to visit the powers of two! They're what you get when you multiply two by itself again and again, like this: 2x2x2x2x2=32, which can also be written like this: $2^5=32$. The powers of two get really big, really fast, because every time the exponent increases by just one, the value doubles! The human population has grown much like this, because each generation multiplies the size of the previous one by some factor. That's what we mean when we say that something is "growing exponentially."

Cubes

You've already visited the square numbers. Now extend the idea of that pattern into three dimensions—it will work just as well. For example, 27, which is the number of small blocks in a 3x3x3 cube like the Rubik's cube below, is one of the cubic numbers. Another way of looking at this is that a cube is anything you can get by multiplying an integer by itself three times. Take a look at the number line. Notice that cubes appear on both sides of 0, but squares all appear to the right of 0. Do you know why?



Highly Composite Numbers

If primes are the atoms of the number world, then highly composite numbers are at the opposite end of the spectrum—the most complex molecules among numbers. To qualify, a number must have more factors than any smaller number has. You'll recognize some familiar numbers in this category- the number of inches in a foot, the number of seconds in a minute, the number of degrees in a circle. These numbers all show up because they're easy to break down into parts, so they're helpful when you want to work with fractions of a foot or slices of a circle.

Pizza Numbers

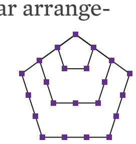
Now we come to a highly practical and potentially tasty pattern. What's the maximum number of pieces you can slice a pizza into with a specific number of straight-line cuts? The pizza numbers are the answer! Note they start out like the powers of two, but soon you get to a spot where you can't quite cut that many pieces. Next time you have a pizza, see if you can cut it into 11 pieces with just four cuts.

Cake Numbers

Cake numbers are just like pizza numbers, only messier. Now you're cutting a cake in three dimensions: you can cut horizontally, vertically, diagonally, or in any other direction as long as you cut straight! As a result, once you're using three cuts or more, the cake numbers are always larger than the pizza numbers. So for example, you can cut a cake into 15 pieces with only four cuts—but some of those pieces would not have much frosting. In fact, one would just be a chunk from the center of the cake!

Pentagonal Numbers

If there are triangular and square numbers, why not pentagonal? In fact, for any regular arrangement of points, you can create a number pattern by counting how many points there are in larger and larger versions of the arrangement.



Guide to the Number Line (Continued)

Constructible Polygon Numbers

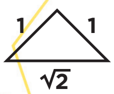
Ancient geometers favored two tools: the compass and the straightedge. They created methods using just these tools for drawing shapes such as equilateral triangles, squares, and pentagons. Millennia later, Gauss (see Triangular Numbers) found a way to draw a regular 17-sided shape, the first polygon construction unknown in classical times. Gauss was so proud of this discovery that he requested the figure be placed on his tombstone. Now we know that only some regular polygons are possible to construct, and this number family tells which.

Tetrahedral Numbers

The tetrahedral numbers extend the triangular numbers into three dimensions, just as the cubes do for the squares. Look at the difference between each pair of tetrahedral numbers—do you recognize the values you find?

Attractions from the Countryside of Real Numbers

$\sqrt{2}$ = 1.41421356...



Some numbers you find in the Real Number countryside are rational—they can be expressed as a fraction with integer numerator and denominator, and their decimal expressions either terminate or enter a repeating pattern. Others, like the square root of 2, are irrational—not equal to any fraction, with decimal expressions that go on forever without repeating. This number is one of the first and simplest irrational numbers encountered in history: the diagonal of a square with sides of length 1.

e = 2.718281828459...

Exponential growth (see Powers of Two) can start from any base, but mathematicians use this one special base called e more than any other. Why? The exponential function e^x has the unique property that its rate of growth is given by the identical expression e^x. This property simplifies calculations done with base e, making it the natural choice for modeling exponential growth in nature.

π = 3.14159265...

Draw a circle, any circle! Carefully measure the circumference, and divide by the diameter. The ratio you get is always the same, no matter how large or small the circle, or where you draw it. For practical and aesthetic reasons, people have been computing this irrational ratio to ever-greater accuracy since the ancient Egyptian times. Modern computing techniques have allowed billions of digits to be determined.

∞ and $-\infty$

What are the outer limits of the Number Line? Where does it end? The true Number Line extends forever in both directions, but alas, our tour must end somewhere. The section shown here is merely a small segment of the whole. So the space beyond the ends is marked with ∞ and $-\infty$, as reminders of the infinite extent of the line that humans can never fully explore.