ORGAN FUNCTION GRINDER

Lesson Plan

GRADE 8

Brought to you by MOMATH

A Letter from the PSEG Foundation

My fascination with energy started at a young age.

The Arab oil embargo of the 1970's sent gasoline prices through the roof and made clear how closely tied our country's foreign policy is to oil interests. I began wondering whether we could produce energy in ways that didn't involve oil, and I wanted to be part of the quest to find the answer.

That passion led me to pursue years of study in the fields of physics and engineering. Graduate degrees in those areas allowed me to take on a variety of fascinating assignments in my career. I served as a research scientist at the Princeton Plasma Physics Lab, a Congressional Science Fellow in the office of U.S. Senator Bill Bradley, and a science, energy, and technology policy advisor to Governor Tom Kean before coming to PSEG where I work every day to create and deliver power responsibly.

This curriculum, developed by the Museum of Mathematics and funded by PSEG, is intended to help young people develop an interest in math and the technical fields–to spark curiosity, stimulate inquiry, and help students down a path of discovery that leads to fulfilling careers.

As issues such as climate change, energy independence, and national security demand increasingly comprehensive and technical solutions, the need for people with knowledge in science, technology, engineering, and math–areas known as the STEM subjects–will continue to grow.

At PSEG, we understand that our country's future depends on developing the insights, creativity, and dynamism of the next generation of innovators. This curriculum is one of many investments we've made in an effort to help young people discover their talents and develop a thirst for knowledge.

A math- and science-savvy workforce will lead the way to innovative technological discovery, a strengthened economy, and thriving new industries. And it is an important part of building a talent pipeline for the energy industry and our country as a whole.

Ralph Izzo
Chairman, CEO and President, PSEG

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Alfred P. Sloan Foundation   PSEG Foundation

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General Instructions for *Math Midway 2 Go*

Math Midway 2 Go (MM2GO) consists of six interactive mathematics exhibits that can travel to schools and other venues. Hands-on activities captivate and engage students, highlighting the wonder of mathematics. These exhibits were designed for use with individuals of all ages, and the mathematical topics they address range from topics in the elementary classroom to college-level mathematics. Students of all ages will benefit from open exploration of the exhibits. At the same time, the exhibits also tie into specific curricular topics for kindergarten through grade 12.

These lesson plans are provided by MoMath to support teachers like you. To help you and your students make the most of your time at *Math Midway 2 Go*, a focus exhibit has been selected for each grade from kindergarten though grade 12. The Grade 8 focus exhibit is the *Organ Function Grinder*.

MM2GO is designed to accommodate one class of up to 36 students at a time.

It is ideal to have only a small group of students at each exhibit while visiting *Math Midway 2 Go*. Break your class into six groups and have them rotate through the exhibits, with one group at each exhibit at a time. Before starting, make sure that students understand basic rules for interacting with the exhibits:

- Walk in the area surrounding the exhibits; don't run.
- Handle the exhibits gently.
- Do not hang or lean on the *Number Line Tightrope*.
- Handle *Ring of Fire* shapes gently.

Ideally, school support staff and/or parent volunteers will be available for the duration of the visit to *Math Midway 2 Go*. These adults can circulate throughout the exhibits, while the classroom teacher remains at the focus exhibit. At the five exhibits that are not the grade-level focus, students can explore and play.

Information about the *Organ Function Grinder*

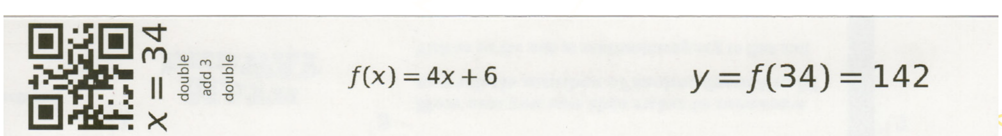
About the exhibit:

The *Organ Function Grinder* comes with an assortment of tickets representing different numbers. Students feed a ticket into the *Organ Function Grinder* as the input to a calculation, turn three dials on the front of the exhibit to select operations to be performed, and turn the crank. The *Organ Function Grinder* plays a song based on the input and the selected operations, calculates the resulting output value, and then delivers a ticket summarizing the input, operations, resulting formula, and output value. Older students can link this experience directly to their study of functions, while younger students can explore rules and how they can be applied to numbers. The music provides another element to explore: how do the different operations change the tunes? What is the connection between the number being processed and the tune?

Why visit the *Organ Function Grinder*?

Secondary students are studying functions. They use prior knowledge of algebra in general and equations in particular to help make sense of functions, and yet functions and equations are not exactly the same, a source of confusion for many secondary students.The *Organ Function Grinder* is a fun way to respond to this learning need. Stressing inputs and outputs, students have the opportunity to link functions to a tangible, intriguing, and memorable exhibit.

Please note–the *Organ Function Grinder* has two settings. It can print tickets with simplified or unsimplified functions. These lesson plans assume that you will use simplified functions with 8th graders. This option takes the three dials and combines them into one function as shown on the ticket below. This format leads to a series of challenging questions. Using only the functions available, how does a ticket end up with x^8? Is there a difference between $2x + 3$ and $2(x+3)$? Which way would you turn the dials to create these functions?



Integrating MM2GO Into Your Unit Plans

Consider the following key questions, class topics, and elements of the Common Core State Standards when considering how to link the *Organ Function Grinder* to the study of mathematics taking place in your classroom.

Key questions inspired by the *Organ Function Grinder*:

- What is a function? How does a function transform an input to an output?
- What happens when a function includes multiple operations? Does order of operations matter in a function?
- With a given input and function, will the output always be the same?

This lesson plan will be useful with the following classes:

- Middle school classes studying functions for the first time
- Pre-algebra classes reviewing functions

Relevant connections to the Common Core State Standards:

Learning Standards

8.F: Define, evaluate, and compare functions.

Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Attend to precision.
- Look for and make use of structure.

Organ Function Grinder Pre-Activity

Description

In this activity, students play a rule guessing game with a partner to mentally practice turning an input into an output according to a function of their choice.

While this activity is designed for use before visiting the *Organ Function Grinder*, the activity can be enjoyed independently of a visit from the Museum of Mathematics' *Math Midway 2 Go*.

Materials

- There are no specialized materials required for this activity, although students may wish to take notes.

Key Terminology

- **Input**
- **Output**
- **Function**
- **Operation**

Conducting the Activity

1. Have each student work with a partner. For ease of directions, have each pair of students identify one partner to be Partner A and the other partner to be Partner B. Explain that students will be playing a game where they guess what rule their partner is using to change one number to another.

2. Partner A will come up with a rule. Give students an example: "My rule is multiply the number by four." Have Partner A think of a rule she will keep as a secret in her head. The rule should use only one operation at this point in the game. Partner B can also think of a rule at this point, which he will use later.

3. Now, tell Partner B that his job is to guess the rule. He will tell Partner A an input number. Partner A will perform the secret calculation in her head and tell him the resulting output number. For example, if the rule were multiply by four and Partner B said "13," then Partner A would respond with "52." Based on this information, Partner B guesses the rule. Possible guesses that Partner B might make based on this example are "Add 39 to the number," or "Subtract the number from 65," or the correct "Multiply the number by four."

Organ Function Grinder Pre-Activity (Continued)

4. If Partner B has guessed incorrectly, he gives a new input, listens to the subsequent output, and guesses again. This repeats until Partner B has correctly guessed Partner A's secret rule.

5. Then, partners switch. Now Partner A has to guess Partner B's secret rule.

6. Once both students have guessed each other's rule, they will work together to come up with a new harder rule, using two operations (for example, double and then add three). When this tricky secret rule is ready, the pair will find another prepared pair and each pair will try to determine the other pair's secret rule. *There may not be sufficient time for all students to determine the tricky, two-step, secret rule. That is okay; students can keep working on it after class is over or during a future period.*

7. Once all students have had some time to try guessing the two-step rule, have a class discussion about the game–what happened?

8. Listen for student comments that relate to functions and explain terms as they come up–**input** is the number before the rule is applied and **output** is the number after the rule is applied. Make sure students learn that this whole process, taking an input and transforming it to an output, is called a **function**. A function is often considered a number machine–one number goes in, a series of operations are performed, and another number comes out.

9. Make sure to stress that the output is completely and uniquely determined by the function and the input. Ask students, if my rule was add four and my input was three, would my output always be seven? This concept–that a given function and a given input always give the same output–is essential to understanding functions.

10. End the class discussion by telling students that they will be exploring a physical machine called the *Organ Function Grinder* during their visit at *Math Midway 2 Go*, where they can adjust the input and the function to deliver a given output.

Organ Function Grinder Pre-Activity (Continued)

Extension: Order of Functions

1. When a function is defined by applying multiple operations, does the order in which you perform them matter? Ask students this question and have a brief discussion.

2. Then, give students a challenge. Tell them that they have three operations and they need to combine them in different ways to get from a given input to a given output. The three operations they can use are "add two," "multiply by three," and "subtract seven."

   Input: 2 Output: 1
   Answer: multiply by 3, add 2, subtract 7 **or** *multiply by 3, subtract 7, add 2*
   Input: 7 Output: 6
   Answer: subtract 7, add 2, multiply by 3 **or** *add 2, subtract 7, multiply by 3*
   Input: 5 Output: 10
   Answer: multiply by 3, subtract 7, add 2 **or** *multiply by 3, add 2, subtract 7*

3. Once students have had time to come to a solution, have them compare with a partner. Did students use the same order with the three functions? Are these the only solutions? Can students identify the operations for which order doesn't matter?

4. Then, challenge the students to use the number 1 as an input and select three functions (add two, multiply by two, and square) in varying order to get four different outputs:

   Input: 1 Output: 36
   Answer: add 2, multiply by 2, square
   Input: 1 Output: 16
   Answer: multiply by 2, add 2, square
   Input: 1 Output: 18
   Answer: add 2, square, multiply by 2
   Input: 1 Output: 4
   Answer: square, multiply by 2, add 2

Organ Function Grinder Pre-Activity (Continued)

5. Once students have solved the problems, have them compare answers with a partner. Which outputs were easy to find? Which were trickier?

6. Ask the class again: "Does order of operations matter in a function?" Have a discussion about what students learned.

Organ Function Grinder Activity

Description

In this activity, students will explore the *Organ Function Grinder*, examining how the machine works and learning about the functional relationship of input to output.

Materials

- There are no specialized materials required for this activity, other than the exhibit itself.

Key Terminology

- **Input**
- **Output**
- **Function**
- **Operation**

Conducting the Activity

1. Allow students to interact with the exhibit at their own pace first. Students can take turns setting the dials, picking an input, and examining the output ticket.

2. Once students have examined the exhibit for a few minutes, bring the group back together. Ask students what they have noticed so far.

3. Have a discussion about the *Organ Function Grinder*. How does it work? What does it do?

4. Depending on your goals for the activity, here are some specific questions to discuss:
    - If you set the dials and don't change them, and then insert the same input again, would you always get the same output? How do you know? This idea–that a given function always takes a given input and turns it into one and only one output–is essential to understanding functions.
    - How does the order of the dials change the output? Are there times when you can change the order of two or three dials and it changes the output? Are there times when changing the order does not change the output? Ask students if they can draw any conclusions about when order does and does not matter.

Organ Function Grinder Activity (Continued)

- Give students the challenge of reaching a goal number. Pick a number and tell them they may use any input and any combination of dials to generate the goal number as the output. Allow students to find multiple solutions, if possible, before changing the goal number to something new.
- Listen to the music. Ask students if they notice any patterns in the music. Try an input with *Organ Function Grinder* set to "add three" three times, and compare that to running it with every dial set to "subtract three." Can your students hear the difference between those operations in the music? Ask them to describe what's happening to the tune when the machine adds three or subtracts three. Can you find an imaginary number? What does the music sound like for an imaginary number? A mathematical musician named Vi Hart composed a tune for each input, and devised ways for it to change based on the selected operations, and also to change when the machine produces certain types of outputs. See if students can figure out some of her musical ideas.

5. End by explaining to students that they will work with an imaginary *Organ Function Grinder* when they are back in the classroom.

Organ Function Grinder Post-Activity

Description

In this activity, students will use an imaginary *Organ Function Grinder* to reach the goal–choosing which operations should be used to transform a given input to a desired output.

While this activity is designed for use after visiting the *Organ Function Grinder*, it can be used with students who have not had the opportunity to experience the Museum of Mathematics' *Math Midway 2 Go*.

Materials

- Attached *Reach the Goal* activity sheet

Key Terminology

- **Input**
- **Output**
- **Function**
- **Operation**

Conducting the Activity

1. Ask students to discuss with a partner: "What do you remember about the *Organ Function Grinder*?" Once students have shared, have a class discussion. When you hear observations related to the terms **input**, **output**, or **function**, take the time to redefine those words as a class.

2. Ensure that students understand that the *Organ Function Grinder* is a machine that takes a given number (**input**), performs a series of operations on it, and then gives a resulting number (**output**). Anything that relates a specific output to any given input is called a **function**.

3. Ask students if they recall the eight settings on the *Organ Function Grinder*. Which operations could it perform? Make a class list of all eight: **double**, **halve**, **add three**, **subtract three**, **square**, **take the square root**, **take the reciprocal**, and **leave alone**.

→

Organ Function Grinder Post-Activity (Continued)

4. Explain to students that today they will be practicing their problem solving skills with an imaginary *Organ Function Grinder*. For each problem, they will be given an input and an output and they will have to figure out how to set the dials on the *Organ Function Grinder* to move from the input to the output. Another way of phrasing this question is to ask, "What function will the *Organ Function Grinder* perform?"

5. Give students time to work on the attached *Reach the Goal* worksheet. Each part is successively more challenging and requires more steps.

6. Remind students that while, in general, functions can use any and all inputs, with the *Organ Function Grinder*, only certain input tickets are available. The possible input numbers are:

   -8, 0, 1, $\sqrt{2}$, 2, 3, π, 4, 7, 8, 9, 10, 13, 15, 16, 18, 28, 32, 34, 42, 53, 64, 81, 97

7. Once students finish Part I and one or two from Part II, have students compare answers in small groups. Did all the students use the same dials? Did they use the same input tickets?

8. Then, have students examine their souvenir tickets from visiting the *Organ Function Grinder*. Ask: "What do the numbers and letters on the ticket tell you?"

9. Explain that $f(x)$ means that function f is performed upon input x. The first equation, which might look like $f(x) = x^8$, is the rule for the function. The second equation, which might show $y = f(8) = 16777216$, shows the input and then the output generated by performing the function.

Organ Function Grinder Post-Activity (Continued)

10. On a separate sheet of paper, have students practice combining the operations they used, and writing these in functional notation. For example, for the first item shown on the *Reach the Goal* worksheet, students would write the following:

$$f(x) = x^2 \qquad y = f(10) = 100$$

11. For each item in Part I, have students practice writing the selected combination of operations in functional notation. They can also try the items in Part II as a challenge, or the items in Part III with the extension below.

12. Finish the class by having students share what they have learned about functions.

Extension: *Reach the Goal* Challenge

Have students work to solve all of the examples from Parts II and III. Some of these examples are quite challenging. You might direct students to begin with the examples that have the greatest number of solutions. If students finish, have them try to find other ways to solve some of the Part II and Part III examples. The attached answer sheet has a full list of the number of solutions for each example.

Organ Function Grinder Post-Activity (Continued)

Extension: Function Challenge

1. Go back to the functions that students wrote for the *Reach the Goal* worksheet. Look at Part III, the first example. Ask students: “How would you write this in functional notation?” Remind them that order probably matters!

2. Establish the input (10), the output (403) and the three dials that are used in sequence (double, square, add three). Ask students to write the expressions that would appear on the *Organ Function Grinder* ticket, were its dials set in this way. Remind students that the *Organ Function Grinder* combines the operations represented by the three dials into a single function. How do students represent all three of these operations in the correct order? Take suggestions from the students, and then check those suggestions. The only way to arrive at 403 is to double 10, square the result, and add three–how can this be represented as an expression?

3. As a class, come to the correct answer:

   $f(x) = 4x2 + 3$ $\qquad$ $y = f(10) = 403$

   Students may start with $f(x) = (2x)2 + 3$. Encourage them to distribute the exponent over the product $2x$, like the computer inside the *Organ Function Grinder* would do.

4. Then, have students practice with other examples from Part II and Part III.

Reach the Goal

Find the input and settings for the dials that will generate the goal output.

Part I: At least two of the dials must be set to "leave alone."

Goal Output	Input	Dial 1	Dial 2	Dial 3
100				
20				
7.5				
49				
8				
0.5				
19				
78				
30				
1/9				

Part II: Use two operations and exactly one dial set to "leave alone."

Goal Output	Input	Dial 1	Dial 2	Dial 3
1/20				
11				
222				
17				
1/4				
400				
-2.5				
-2				
84				
25				

Part III: Use three operations and no dials set to "leave alone."

Goal Output	Input	Dial 1	Dial 2	Dial 3
403				
3 1/4				
124				
4/49				
17.5				
-10				
1/4				
27				
3 1/9				
9997				

Reach the Goal

Answers (there may be other possible answers; the number of solutions is listed)

Part I

Goal Output	Input	Dial 1	Dial 2	Dial 3	How many other solutions?
100	10	Square	Leave alone	Leave alone	1
20	10	Double	Leave alone	Leave alone	0
7.5	15	Halve	Leave alone	Leave alone	0
49	7	Square	Leave alone	Leave alone	0
8	64	Square root	Leave alone	Leave alone	3
0.5	1	Halve	Leave alone	Leave alone	1
19	16	Add three	Leave alone	Leave alone	0
78	81	Subtract three	Leave alone	Leave alone	0
30	15	Double	Leave alone	Leave alone	0
1/9	9	Invert	Leave alone	Leave alone	0

Part II

Goal Output	Input	Dial 1	Dial 2	Dial 3	How many other solutions?
1/20	10	Double	Invert	Leave alone	1
11	4	Double	Add three	Leave alone	4
222	15	Square	Subtract three	Leave alone	0
17	10	Double	Subtract three	Leave alone	2
1/4	2	Invert	Square	Leave alone	11
400	10	Double	Square	Leave alone	0
-2.5	-8	Add three	Halve	Leave alone	2
-2	2	Halve	Subtract three	Leave alone	7
84	9	Square	Add three	Leave alone	0
25	2	Add three	Square	Leave alone	4

Part III

Goal Output	Input	Dial 1	Dial 2	Dial 3	How many other solutions?
403	10	Double	Square	Add three	0
3 3/4	2	Invert	Square	Add three	15
124	8	Add three	Square	Add three	3
4/49	7	Halve	Invert	Square	2
17.5	16	Double	Add three	Halve	2
-10	1	Subract three	Subtract three	Double	2
1/4	4	Halve	Invert	Square	69
27	9	Add three	Double	Add three	4
3 1/9	3	Invert	Square	Add three	5
9997	10	Square	Square	Subract three	1

Note that some instances of multiple solutions result from operations that commute with each other (i.e., produce the same result in either order) and some do not.