NCTM Standards and the *Math Midway*

The exhibits that make up the *Math Midway* are exploratory, hands-on, interactive, and open-ended in nature. This design allows educators to incorporate a visit to the *Math Midway* into their standards-based curriculum and seamlessly adapt content to the grade level of their students. The diversity of the standards addressed by the *Math Midway* ensures that exhibits will be relevant to those topics currently being covered in the classroom.

What follows is a discussion of how the standards from the National Council of Teachers of Mathematics can be addressed through interaction with the exhibits of the *Math Midway*. Because the exhibition will be traveling nationally, the exhibits are correlated to the NCTM standards, but they should also be easily aligned to any state's curriculum standards. The first section addresses the *Math Midway* as a rich vehicle to develop the Process Standards of the NCTM. These standards apply to all students from prekindergarten through grade 12, focusing on Problem Solving, Reasoning and Proof, Communication, Connections, and Representations. Virtually any *Math Midway* exhibit could facilitate exploration of these concepts.

The next section, focusing on the Content Standards for grades 3-12, contains a description of each *Math Midway* exhibit and a list of the standards supported by that exhibit. The parenthetical numbers are the grade levels associated with that particular standard. This list is not comprehensive, however, because creative educators can find limitless ways to use these exhibits to create excitement about mathematics while helping their students to learn new content or master new skills.

NCTM Process Standards

Many of the *Math Midway* exhibits pose a particular problem for a student to solve, and this problem can often be solved with any number of strategies. Many of the exhibits also offer opportunities to work cooperatively, where communication is at the forefront of a student's experience. Other exhibits are more open-ended in nature, which allows for the creation of both conjectures to test and representations to record and analyze observations. The exhibits also show interesting contexts where mathematical ideas can be highlighted, encouraging students to find their own connections between mathematics and their everyday life. Ultimately, all of these Process Standards can be explored through a visit to the *Math Midway*. Please note that the exhibits mentioned in the "Opportunities" sections are just examples; many more exhibits can be used to support these standards.

Problem Solving
Instructional programs from prekindergarten through grade 12 should enable all students to—
- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

Opportunities for Problem Solving

Developing strategies and processes for problem solving can be the focus at exhibits such as Pirate X and Lady Y and A-Maze-ing Math, where students can learn to solve an equation step-by-step or solve a maze by working backwards. Exhibits like Traveling Carnival allow students to explore problems that have a real application outside of mathematics.

Reasoning and Proof
Instructional programs from prekindergarten through grade 12 should enable all students to—
- Recognize reasoning and proof as fundamental aspects of mathematics
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs
- Select and use various types of reasoning and methods of proof

Opportunities for Reasoning and Proof

Many exhibits provide students with opportunities to make and test their mathematical conjectures. At Math Unleashed, students can deduce which post will free their tangled dog and see if they're correct by removing that post. At Roller Graphicoaster, they can reason about the best shape for the

fastest roller coaster track and run experiments to test their guesses. At Ring of Fire, students can conjecture what the cross section of a three-dimensional solid will be before the ring of laser light shows them the answer.

Communication
Instructional programs from prekindergarten through grade 12 should enable all students to—
- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely

Opportunities for Communication

Any time students are working together at these exhibits, they can communicate their strategies and thinking with peers and teachers. Exhibits like Amazing Acrobats and Polyhedral Puzzle Plaza create perfect opportunities to work together to construct large geometrical objects. Students can also work in teams at exhibits like Universal Wheel of Chance to design their own new games or activities.

Connections
Instructional programs from prekindergarten through grade 12 should enable all students to—
- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

Opportunities for Connections

Some of the exhibits in the *Math Midway* are connected through explorations of similar ideas—for example, A-Maze-ing Math and Traveling Carnival both deal with finding a good path through a network, and Coffee Cup Curves and Mirror Morph deal with reflections. Many exhibits also connect to contexts outside of mathematics, such as Pedal on the Petals, Traveling Carnival, and Roller Graphicoaster.

Representation

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena

Opportunities for Representation

The Organ Function Grinder is a great exhibit to connect the various aspects of functions, from the words on the exhibit itself to the algebraic notation that accompanies the output of the organ grinder. Exhibits like Pirate X and Lady Y allow students to model their setup of the pirate ship, coins, and characters with an algebraic equation or inequality.

Pedal on the Petals

Standards
Geometry
- Create and describe mental images of objects, patterns, and paths (3-5)
- Describe location and movement using common language and geometric vocabulary (3-5)
- Recognize and apply geometric ideas to problems that arise in the classroom or in everyday life (3-5) or to areas outside the mathematics classroom such as art and science (6-8).

Number Line Tightrope

Standards

Number and Operation

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems (3-5)
- Explore numbers less than 0 by extending the number line and through familiar applications (3-5)
- Recognize equivalent representations for the same number and generate them by decomposing and composing numbers (3-5)
- Develop understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers (3-5)
- Describe classes of numbers according to characteristics such as the nature of their factors (3-5)
- Use factors, multiples, prime factorization, and relatively prime numbers to solve problems (6-8)
- Develop meaning for integers and represent and compare quantities with them (6-8)
- Compare and contrast the properties of numbers and number systems, including the rational and real numbers, and understand complex numbers as solutions to quadratic equations that do not have real solutions (9-12)

Algebra

- Understand patterns, relations, and functions (3-5)
- Generalize patterns using explicitly defined and recursively defined functions (9-12)

Organ Function Grinder

<u>Standards</u>
Algebra
- Represent and analyze patterns and functions, using words, tables, and graphs (3-5)
- Identify such properties as commutativity, associativity, and distributivity and use them to compute with whole numbers (3-5)
- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules (6-8)
- Relate and compare different forms of representation for a relationship (6-8)
- Develop an initial conceptual understanding of different uses of variables (6-8)
- Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships (6-8)
- Recognize and generate equivalent forms for simple algebraic expressions and solve linear equations (6-8)
- Model and solve contextualized problems using various representations, such as graphs, tables, and equations (6-8)

Number and Operation
- Use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals (6-8)
- Understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems (6-8)
- Develop and use strategies to estimate the results of rational number computations and judge the reasonableness of the results (6-8)
- Judge the effects of such operations as multiplication, division, and computing powers and roots on the magnitudes of quantities (9-12)
- Understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more complicated symbolic expressions (9-12)
- Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology (9-12)

Mathematical Monkey Mat

Standards

Geometry

- Predict and describe the results of sliding, flipping, and turning two-dimensional shapes (3-5)
- Describe a motion or a series of motions that will show that two shapes are congruent (3-5)
- Identify and describe line and rotational symmetry in two- and three-dimensional shapes and designs (3-5)
- Examine the congruence, similarity, and line or rotational symmetry of objects using transformations (6-8)
- Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life (6-8)
- Analyze properties and determine attributes of two- and three-dimensional objects (9-12)
- Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them (9-12)
- Understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices (9-12)
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture (9-12)

Polyhedral Puzzle Plaza

<u>Standards</u>
Geometry
- Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes (3-5)
- Classify two- and three-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids (3-5)
- Investigate, describe, and reason about the results of subdividing, combining, and transforming shapes (3-5)
- Build and draw geometric objects (3-5)
- Create and describe mental images of objects, patterns, and paths (3-5)
- Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties (6-8)
- Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects (6-8)
- Use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume (6-8)
- Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life (6-8)
- Analyze properties and determine attributes of two- and three-dimensional objects (9-12)
- Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them (9-12)
- Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools (9-12)
- Visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections (9-12)
- Use geometric models to gain insights into, and answer questions in, other areas of mathematics (9-12)
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture (9-12)

Traveling Carnival

Standards

Number and Operation

- Develop and use strategies to estimate the results of whole and rational number computations and to judge the reasonableness of such results (3-5), (6-8)

Measurement

- Understand such attributes as length, area, weight, volume, and size of angle and select the appropriate type of unit for measuring each attribute (3-5)
- Understand the need for measuring with standard units and become familiar with standard units in the customary and metric systems (3-5)
- Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision (6-8)

Geometry

- Use visual tools such as networks to represent and solve problems (6-8)
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture (9-12)

A-Maze-ing Math

Standards
Geometry
- Create and describe mental images of objects, patterns, and paths (3-5)
- Use vertex-edge graphs to model and solve problems (9-12)

Miles of Tiles

Standards
Geometry
- Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes (3-5)
- Classify two- and three-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids (3-5)
- Investigate, describe, and reason about the results of subdividing, combining, and transforming shapes (3-5)
- Describe location and movement using common language and geometric vocabulary (3-5)
- Predict and describe the results of sliding, flipping, and turning two-dimensional shapes (3-5)
- Identify and describe line and rotational symmetry in two- and three-dimensional shapes and designs (3-5)
- Build and draw geometric objects (3-5)
- Create and describe mental images of objects, patterns, and paths (3-5)
- Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties (6-8)
- Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects (6-8)
- Describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling (6-8)
- Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life (6-8)
- Analyze properties and determine attributes of two- and three-dimensional objects (9-12)
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture (9-12)

Plant the Daisy

Standards

Geometry

- Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life (6-8)
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture (9-12)

Math Unleashed

Standards

Algebra

- Model and solve contextualized problems using various representations, such as graphs, tables, and equations (6-8)
- Use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts (9-12)
- Draw reasonable conclusions about a situation being modeled (9-12)

Geometry

- Create and describe mental images of objects, patterns, and paths (3-5)
- Recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life (3-5)
- Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life (6-8)

Amazing Acrobats

<u>Standards</u>
Geometry

- Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes (3-5)
- Classify two- and three-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids (3-5)
- Investigate, describe, and reason about the results of subdividing, combining, and transforming shapes (3-5)
- Predict and describe the results of sliding, flipping, and turning two-dimensional shapes (3-5)
- Identify and describe line and rotational symmetry in two- and three-dimensional shapes and designs (3-5)
- Build and draw geometric objects (3-5)
- Recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life (3-5)
- Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties (6-8)
- Describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling (6-8)
- Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life (6-8)
- Analyze properties and determine attributes of two- and three-dimensional objects (9-12)
- Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them (9-12)
- Visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections (9-12)
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture (9-12)

Ring of Fire

<u>Standards</u>
Geometry
- Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes (3-5)
- Classify two- and three-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids (3-5)
- Investigate, describe, and reason about the results of subdividing, combining, and transforming shapes (3-5)
- Identify and describe line and rotational symmetry in two- and three-dimensional shapes and designs (3-5)
- Build and draw geometric objects (3-5)
- Recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life (3-5)
- Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties (6-8)
- Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life (6-8)
- Analyze properties and determine attributes of two- and three-dimensional objects (9-12)
- Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them (9-12)
- Visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections (9-12)
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture (9-12)

Roller Graphicoaster

<u>Standards</u>

Algebra
- Identify and describe situations with constant or varying rates of change and compare them (3-5)
- Model and solve contextualized problems using various representations, such as graphs, tables, and equations (6-8)
- Draw reasonable conclusions about a situation being modeled (9-12)
- Approximate and interpret rates of change from graphical and numerical data (9-12)

Geometry
- Make and use coordinate systems to specify locations and to describe paths (3-5)
- Find the distance between points along horizontal and vertical lines of a coordinate system (3-5)
- Create and describe mental images of objects, patterns, and paths (3-5)
- Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates (9-12)

Data Analysis
- Collect data using observations, surveys, and experiments (3-5)
- Represent data using tables and graphs such as line plots, bar graphs, and line graphs (3-5)
- Use measures of center, focusing on the median, and understand what each does and does not indicate about the data set (3-5)
- Propose and justify conclusions and predictions that are based on data and design studies to further investigate the conclusions or predictions (3-5)

Mirror Morph

Standards

Geometry

- Investigate, describe, and reason about the results of subdividing, combining, and transforming shapes (3-5)
- Create and describe mental images of objects, patterns, and paths (3-5)
- Recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life (3-5)
- Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life (6-8)
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture (9-12)
- Analyze properties and determine attributes of two- and three-dimensional objects (9-12)

Coffee Cup Curves

Standards
Geometry
- Create and describe mental images of objects, patterns, and paths (3-5)
- Recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life (3-5)
- Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life (6-8)
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture (9-12)
- Analyze properties and determine attributes of two- and three-dimensional objects (9-12)

Funny Face

Standards
Geometry
- Investigate, describe, and reason about the results of subdividing, combining, and transforming shapes (3-5)
- Create and describe mental images of objects, patterns, and paths (3-5)
- Recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life (3-5)
- Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life (6-8)
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture (9-12)

The Mysterious Harmonograph

Standards

Algebra
- Investigate how a change in one variable relates to a change in a second variable (3-5)
- Identify and describe situations with constant or varying rates of change and compare them (3-5)
- Develop an initial conceptual understanding of different uses of variables (6-8)
- Approximate and interpret rates of change from graphical and numerical data (9-12)

Geometry
- Create and describe mental images of objects, patterns, and paths (3-5)
- Recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life (3-5)
- Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life (6-8)
- Approximate and interpret rates of change from graphical and numerical data (9-12)
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture (9-12)

Data Analysis
- Collect data using observations, surveys, and experiments (3-5)
- Use conjectures to formulate new questions and plan new studies to answer them (6-8)

Universal Wheel of Chance

Standards
Data Analysis

- Describe events as likely or unlikely and discuss the degree of likelihood using such words as certain, equally likely, and impossible (3-5)
- Predict the probability of outcomes of simple experiments and test the predictions (3-5)
- Understand that the measure of the likelihood of an event can be represented by a number from 0 to 1 (3-5)
- Understand and use appropriate terminology to describe complementary and mutually exclusive events (6-8)
- Use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations (6-8)
- Compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models (6-8)
- Use conjectures to formulate new questions and plan new studies to answer them (6-8)
- Use simulations to construct empirical probability distributions (9-12)
- Understand the concepts of conditional probability and independent events (9-12)
- Understand how to compute the probability of a compound event (9-12)

Pirate X and Lady Y

Standards
Algebra

- Describe, extend, and make generalizations about geometric and numeric patterns (3-5)
- Represent and analyze patterns and functions, using words, tables, and graphs (3-5)
- Represent the idea of a variable as an unknown quantity using a letter or a symbol (3-5)
- Express mathematical relationships using equations (3-5)
- Model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions (3-5)
- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules (6-8)
- Relate and compare different forms of representation for a relationship (6-8)
- Develop an initial conceptual understanding of different uses of variables (6-8)
- Recognize and generate equivalent forms for simple algebraic expressions and solve linear equations (6-8)
- Model and solve contextualized problems using various representations, such as graphs, tables, and equations (6-8)
- Generalize patterns using explicitly defined and recursively defined functions (9-12)
- Interpret representations of functions of two variables (9-12)
- Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases (9-12)
- Use symbolic algebra to represent and explain mathematical relationships (9-12)
- Use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts (9-12)
- Draw reasonable conclusions about a situation being modeled (9-12)

Magician and the Moon

<u>Standards</u>

Geometry

- Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes (3-5)
- Make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions (3-5)
- Create and describe mental images of objects, patterns, and paths (3-5)
- Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects (6-8)

Three Ring Circleous

<u>Standards</u>
Geometry
- Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes (3-5)
- Classify two- and three-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids (3-5)
- Make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions (3-5)
- Identify and describe line and rotational symmetry in two- and three-dimensional shapes and designs (3-5)
- Create and describe mental images of objects, patterns, and paths (3-5)
- Identify and build a three-dimensional object from two-dimensional representations of that object (3-5)
- Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties (6-8)
- Examine the congruence, similarity, and line or rotational symmetry of objects using transformations (6-8)
- Use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume (6-8)
- Analyze properties and determine attributes of two- and three-dimensional objects (9-12)
- Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them (9-12)
- Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools (9-12)
- Visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections (9-12)