Coffee Cup Curves

Brief Description:
This exhibit consists of a circular table with a printed top, a bright spotlight, and flexible plastic mirrors. Students explore the curves of light that are created by the reflections of straight lines against a curved surface.

Objectives:
Students will attempt to solve the different shape challenges offered by the exhibit, or they can create their own interesting reflection patterns.

Links to Websites:
http://mathmidway.org/Training/coffee.php

http://www.math.admu.edu.ph/~jumela/EDSC320/Conic%20Sections.pdf

http://www.mathinscience.info/public/conic_sections/folding_conic_sections.pdf

http://www.mathcats.com/crafts/stringart.html

Vocabulary:

Caustic curves	Circle
Circumference	Conic sections
Directrix	Ellipse
Focus	Hyperbola
Nephroid	Parabola
Spirograph	Tangent

Before:
- (*Levels 2, 3*) Straight lines can be used to construct conic sections (parabola, ellipse, circle, hyperbola). The following websites provide instructions and explanations for the "how-to."

  http://www.math.admu.edu.ph/~jumela/EDSC320/Conic%20Sections.pdf

  http://www.mathinscience.info/public/conic_sections/folding_conic_sections.pdf

- (*Levels 1, 2*) The Coffee Cup Curves exhibit shows you how you can make curves out of straight lines. In the case of the exhibit, these straight lines are the rays of light emanating from the lamp above the exhibit. You can demonstrate this idea with your students using string art. You can make these pictures using paper and a straight edge, or if you have the materials, you can use colored thread, straight pins, and foam.
  A good website for the basic string art patterns is here:
  http://www.mathcats.com/crafts/stringart.html

- (*Level 3*) For a more advanced string art pattern, you can make a nephroid shape, which is one of the curves you will be able to see at the Coffee Cup Curves exhibit. (See below for instructions.)

The nephroid is the path followed by a point on the circumference of a circle of radius one unit as it rolls around the circumference of a circle of radius two units. It is formed here from its tangents by laying N points equally on a circle, and joining point n to the point ($3n$ + N/2) mod N. The nephroid may be demonstrated using spirograph techniques, using either the toy or a computer application.

The nephroid may also be formed by rolling a circle of radius r, around the outside of a circle of radius $2r$. *What curve does rolling a circle of radius r inside a circle of radius 2r form?*
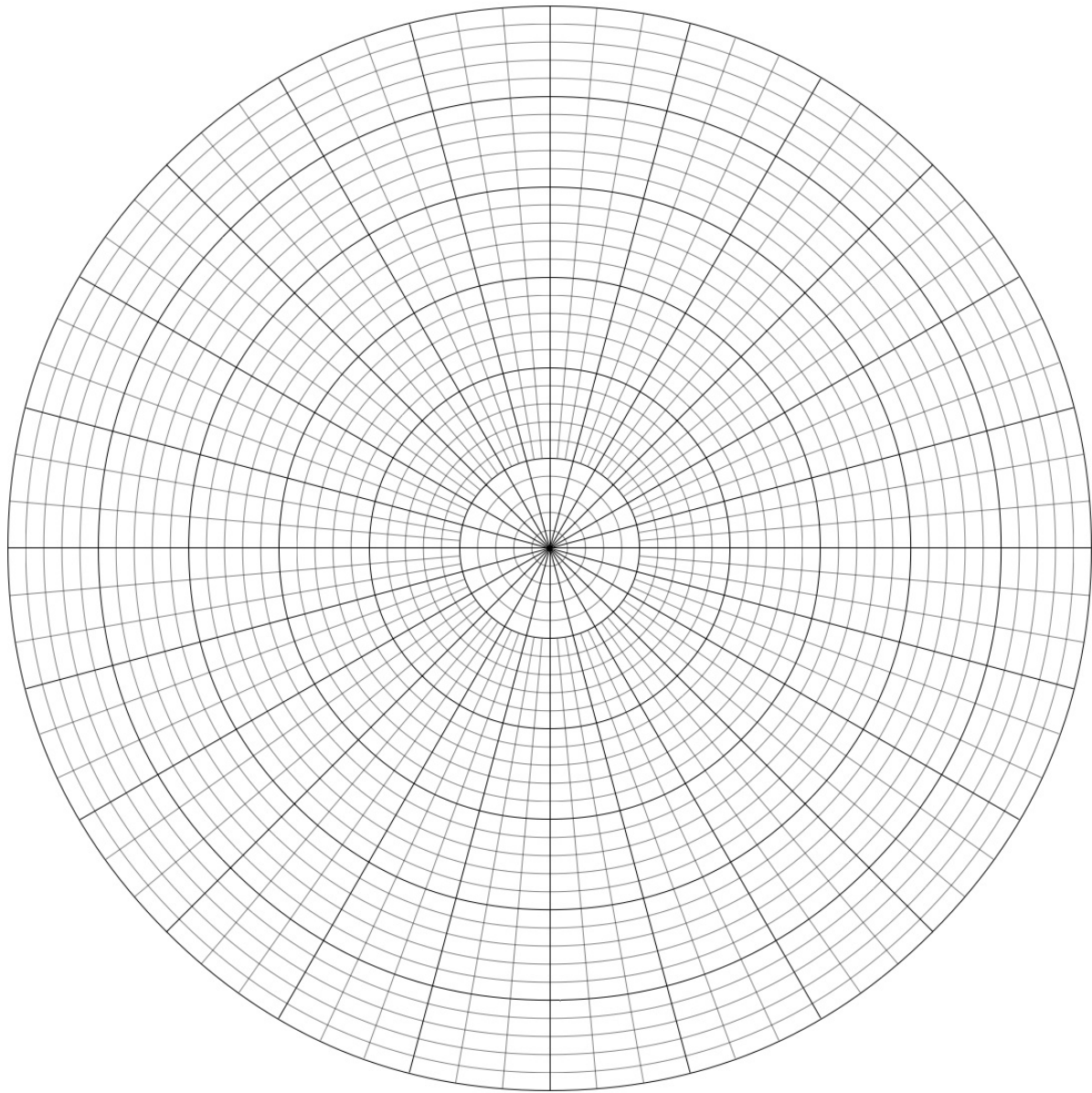
Drawing a Nephroid

Draw a point at each location on the circle where a radius intersects with the outer circle. Number these points starting with 1. There should be 72 of these points.

To figure out which points to connect to make the nephroid shape, fill out this chart. The first several are done as an example. For each number, multiply it by three and then add 36. Put this number in the second column. The third column is found by dividing the number in the second column by 72, and taking the remainder. This can be done very nicely in a spreadsheet such as Excel. The second column may be omitted since you can use the formula: *=mod(3*A1+36,72)* in column B when column A are the integers from 1 to 72.

Connect the numbered dot with the number in the remainder column with a straight line. The numbers 12, 36, and 60 need to connect to the number 0, but 0 is not one of the numbered points. Just ignore these since each of these numbers will be connected to the following sets of numbers: 12 connects to 16, 40, and 64, 36 connects to 24, 48, and 72, and 60 connects to 8, 32, and 56. You might also notice that all numbers divisible by 3 connect to three other numbers.
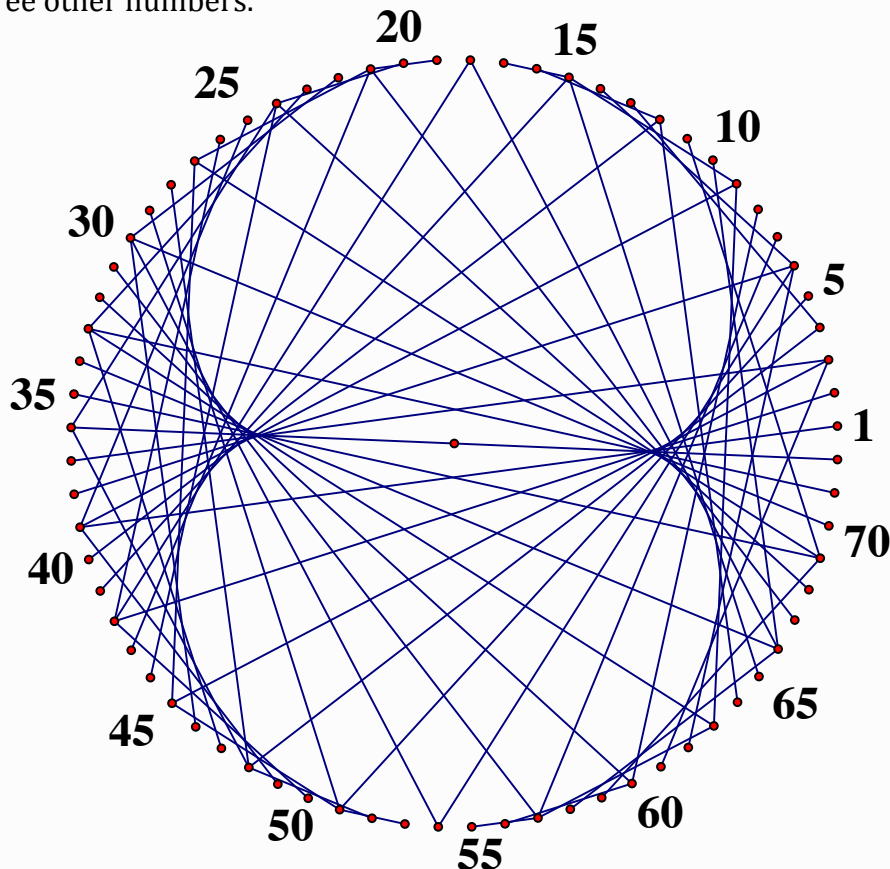
Number (n)	3n+36 (b)	Remainder of b divided by 72	Number (n)	3n+36 (b)	Remainder of b divided by 72
1	39	39	37	147	3
2	42	42	38	150	6
3			39		
4			40		
5			41		
6			42		
7			43		
8			44		
9			45		
10			46		
11			47		
12	72	0	48		
13			49		
14			50		
15			51		
16			52		
17			53		
18	90	18	54		
19			55		
20			56		
21			57		
22			58		
23			59		
24			60		0
25			61		
26			62		
27			63		
28			64		
29			65		
30			66		
31			67		
32			68		
33			69		
34			70		
35			71		
36		0	72		

During:

- ⊙ (*Levels 1, 2, 3*) Explore the activities suggested by the graphic on the Coffee Cup Curves table.

After:

- ⊙ Look for examples of nephroids in the real world