Pedal on the Petals

Brief Description:

There are two square-wheeled trikes, one for adults and one child-sized. The circular track is shaped like a flower, with ridges that have catenary cross-sections. Each wheel on the trike is a different size, in proportion to its distance from the center, so each turn of each wheel covers the same angular distance along the flower. One trike has the small wheel on the right, and will go around the track clockwise, while the other has the opposite orientation.

Objectives:

Discover the catenary curve and how this shape makes it possible for square wheels to roll while maintaining a fixed height center. At this exhibit, after taking a smooth ride on the square-wheeled tricycle, students can match other unusual wheel shapes with their corresponding tracks, honing spatial visualization skills and making surprising discoveries.

Links to websites:

http://mathmidway.org/Training/pedals.php

http://www.youtube.com/watch?v=LgbWu8zJubo&feature=fvw

http://teachers.sduhsd.k12.ca.us/abrown/Activities/Matching/answers/Catenary.htm

http://demonstrations.wolfram.com/RegularPolygonRollingOnACatenary/

Vocabulary:

Fibonacci number
Golden Ratio
Radius
Regular Polygon
Square
The Catenary Curve

Before:

- (*Levels 1, 2 and 3*) Show the following video of a square-wheeled tricycle: http://www.youtube.com/watch?v=LgbWu8zJubo&feature=fvw, and ask the students why they think the ride is so smooth. Discuss the reasons and explain that they will get a chance to ride a square-wheeled tricycle.

- (*Level 1)* Read The Greedy Triangle and design a worksheet for the children to find the different shapes mentioned in the book at the museum during their visit. [Math Midway has a suggested lesson plan in its brochure Educators Math Midway Museum Activities.]

- (*Levels 2 and 3*) Discuss the explanation of a catenary on the following website: http://teachers.sduhsd.k12.ca.us/abrown/Activities/Matching/answers/Catenary.htm

- (*Level 2, 3*) Create a set of shapes that students cut out and match up with a set of possible "roads" for them to roll on. Students would have to match the wheel with the road by rolling the cutout on the drawing of the road they believe best fits it. This is modeled on the website: http://demonstrations.wolfram.com/RegularPolygonRollingOnACatenary/. It is worthwhile to download the Mathematica Player in order to manipulate the polygons. The physical activity combines nicely with the mathematical construct. This activity should be

prepared in advance of the visit by printing out a group of roadways and regular polygons which can be found by returning to the Educator's Guide page and clicking on the Student Handout called Roads and Wheels.

- (*Level 3*) Discover the difference between a parabola and a catenary, look at how they're different on a graphing calculator.

  Parabola: $y = x^2$

  Catenary: $y = \cosh(x)$ or $y = \frac{\left(e^{x} + e^{-x}\right)}{2}$

- Provide illustrations for each of the terms in the vocabulary list.

During:

- (*Level 1, 2, 3*) While students are waiting for their turn to ride a square-wheeled tricycle, introduce them to the idea of the catenary curve. Not only is this the shape of the curves on the sunflower track, but it is a curve that can be seen elsewhere in the world. Look at the yellow chains that hang between the stanchions around the exhibit. The curved shape that the chain makes as it hangs between two stanchions is a catenary curve. If you can imagine that curve flipped upside down, you will have the shape of the track. You can make this shape yourself using any length of cord, rope, or chain hanging between your hands.

- Provide wax-coated string or tape measures for students to measure the side of the square wheel and the length of the arc of the track under the wheel from one trough to the next. *Why do the measures have to be the same?*

- Investigations: Create a treasure hunt from the vocabulary at the Museum.
  Can you discover other catenaries in the Math Midway? In the Real World?
  What is the difference between getting a smooth ride from circular wheels on a flat roadway and one from square-wheels on a "catenary" roadway?
  Why are the wheels on the square-wheeled trikes different sizes?

After:

- (Math Fair Project) Create your own catenary curves with clay and square rod.
  Measure the length of one side of the square rod.
  Cut out strips of paper the size of one side of the square rod.
  Place strips of paper on clay to "mold" the catenary curves.

- (*Level 3*) There can be a discussion about the angle of the polygonal wheel and the angle formed by the tangents at the point where 2 catenary sections meet.

- Investigations: (*Level 2, 3*)
  What would happen if the wheel had a different number of sides? Try a pentagon and a triangle.

- Find pictures on the internet of catenaries in the real world.

- (Levels 1, 2, 3) Worksheets for replicating regular polygons and catenary curves will be on http://momath.org/ as pdf files.