Teacher's Guide

for Visiting the Math Midway

The Museum of Mathematics

For more information about the Math Midway and the Museum of Mathematics, please see momath.org and mathmidway.org. In addition, you may access a set of lesson plans called Math_Midway_Class_Activities.doc for a select group of activities to help make your visit to the museum an awesome event.

Table of Contents

Welcome to the Math Midway!

This guide is designed for classroom teachers who will be leading a class visit to the Math Midway. It provides ideas for activities to prepare the class before the trip, to do with students during the trip, and to follow up back in the classroom after the trip.

A visit to the Midway will give you and your students a chance to be surprised by the breadth of mathematics. You will see how mathematics can be an experimental science, how it can be a lens through which to view a situation, and perhaps most importantly, how math can be a fun activity accessible to everyone.

How To Use This Educator's Guide

Each exhibit's section has examples of activities to do with your students before, during, and after an interaction with that exhibit. You may want to focus on particular exhibits during your visit to the Math Midway, and follow through with the activities completely. You may also wish to see what interests are sparked among your students during the visit, and pick follow-up activities that match their interests. You may choose to do activities that follow closely to your curriculum, or activities that provide enrichment outside of the normal curriculum. Some activities and investigations are labeled by levels: 1= beginner or elementary (generally pre-K-6), 2 = intermediate or standard (generally 4-9), and 3 = advanced (generally 8-college). We've tried to make these activities as diverse as possible to fit the variety of ways classroom teachers may incorporate their visit into their school year. Key questions to elicit thoughtful responses are in italics.

Teachers should visit http://mathmidway.org/Training/ (note that the capital "T" is necessary) to view an introduction to each of the exhibits described in this document. For your convenience this document will be on the web to enable you to follow the links without typing the URL's yourself. If you have other ideas or suggestions, let us know. We would love for this guide to be a living document of how students interact best with the Math Midway.

Pedal on the Petals

Brief Description:
There are two square-wheeled trikes, one for adults and one child-sized. The circular track is shaped like a flower, with ridges that have catenary cross-sections. Each wheel on the trike is a different size, in proportion to its distance from the center, so each turn of each wheel covers the same angular distance along the flower. One trike has the small wheel on the right, and will go around the track clockwise, while the other has the opposite orientation.

Objectives:
Discover the catenary curve and how this shape makes it possible for square wheels to roll while maintaining a fixed height center. At this exhibit, after taking a smooth ride on the square-wheeled tricycle, students can match other unusual wheel shapes with their corresponding tracks, honing spatial visualization skills and making surprising discoveries.

Links to websites:
http://mathmidway.org/Training/pedals.php

http://www.youtube.com/watch?v=LgbWu8zJubo&feature=fvw

http://teachers.sduhsd.k12.ca.us/abrown/Activities/Matching/answers/Catenary.htm

http://demonstrations.wolfram.com/RegularPolygonRollingOnACatenary/

Vocabulary:
Fibonacci number	Golden Ratio
Radius	Regular Polygon
Square	The Catenary Curve

Before:
- (*Levels 1, 2 and 3*) Show the following video of a square-wheeled tricycle: http://www.youtube.com/watch?v=LgbWu8zJubo&feature=fvw, and ask the students why they think the ride is so smooth. Discuss the reasons and explain that they will get a chance to ride a square-wheeled tricycle.

- (*Level 1)* Read The Greedy Triangle and design a worksheet for the children to find the different shapes mentioned in the book at the museum during their visit. [Math Midway has a suggested lesson plan in its brochure Educators Math Midway Museum Activities.]

- (*Levels 2 and 3*) Discuss the explanation of a catenary on the following website: http://teachers.sduhsd.k12.ca.us/abrown/Activities/Matching/answers/Catenary.htm

- (*Level 2, 3*) Create a set of shapes that students cut out and match up with a set of possible "roads" for them to roll on. Students would have to match the wheel with the road by rolling the cutout on the drawing of the road they believe best fits it. This is modeled on the website: http://demonstrations.wolfram.com/RegularPolygonRollingOnACatenary/. It is worthwhile to download the Mathematica Player in order to manipulate the polygons. The physical activity combines nicely with the mathematical construct. This activity should be prepared in advance of the visit by printing out a group of roadways and regular polygons which can be found by returning to the Educator's Guide page and clicking on the Student Handout called Roads and Wheels.

- (*Level 3*) Discover the difference between a parabola and a catenary, look at how they're different on a graphing calculator.

  Parabola: $y = x^2$

  Catenary: $y = \cosh(x)$ or $y = \frac{(e^x + e^{-x})}{2}$

- Provide illustrations for each of the terms in the vocabulary list.

During:

- (*Level 1, 2, 3*) While students are waiting for their turn to ride a square-wheeled tricycle, introduce them to the idea of the catenary curve. Not only is this the shape of the curves on the sunflower track, but it is a curve that can be seen elsewhere in the world. Look at the yellow chains that hang between the stanchions around the exhibit. The curved shape that the chain makes as it hangs between two stanchions is a catenary curve. If you can imagine that curve flipped upside down, you will have the shape of the track. You can make this shape yourself using any length of cord, rope, or chain hanging between your hands.

- Provide wax-coated string or tape measures for students to measure the side of the square wheel and the length of the arc of the track under the wheel from one trough to the next. *Why do the measures have to be the same?*

- Investigations: Create a treasure hunt from the vocabulary at the Museum.
  Can you discover other catenaries in the Math Midway? In the Real World?
  What is the difference between getting a smooth ride from circular wheels on a flat roadway and one from square-wheels on a "catenary" roadway?
  Why are the wheels on the square-wheeled trikes different sizes?

After:

- (Math Fair Project) Create your own catenary curves with clay and square rod.
  Measure the length of one side of the square rod.
  Cut out strips of paper the size of one side of the square rod.
  Place strips of paper on clay to "mold" the catenary curves.

- (*Level 3*) There can be a discussion about the angle of the polygonal wheel and the angle formed by the tangents at the point where 2 catenary sections meet.

- Investigations: (*Level 2, 3*)
  What would happen if the wheel had a different number of sides? Try a pentagon and a triangle.

- Find pictures on the internet of catenaries in the real world.

- (Levels 1, 2, 3) Worksheets for replicating regular polygons and catenary curves will be on http://momath.org/ as pdf files.

Directions to Roads and Wheels

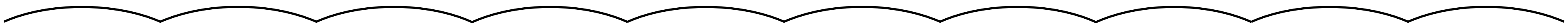
Download the file Roads and Wheels. If you have a heavy stock paper print the Wheels on that paper. The Roads page can be printed on any type of paper. Either you or your students should cut out the wheels and then have the children match the wheel to the correct road. Each wheel should roll smoothly on only one of the road surfaces. Try not to let the wheel slip as you rotate it along the road.

Answers: The square fits on Road A, the pentagon on Road B, the hexagon on Road C, the heptagon on Road D and the octagon on Road E.

Road *A*

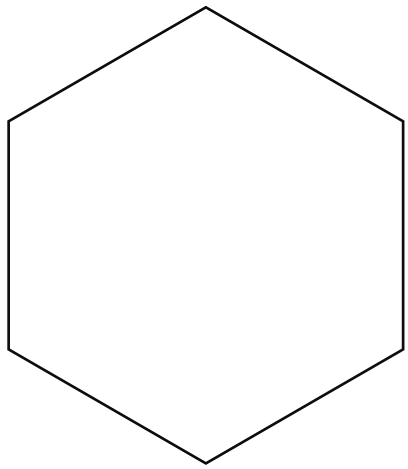
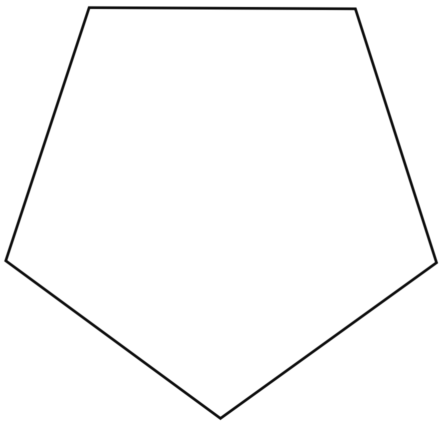
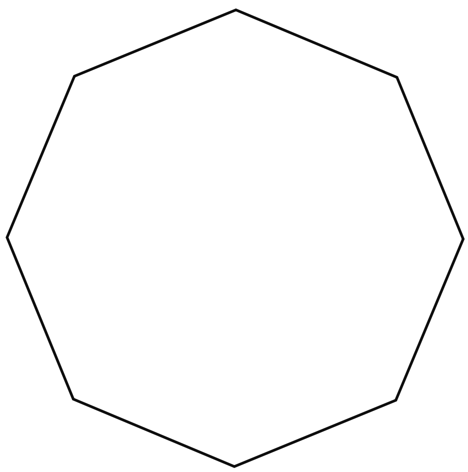
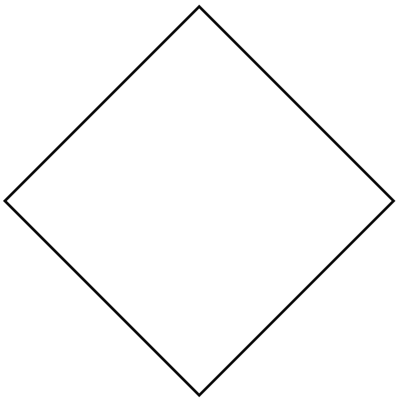
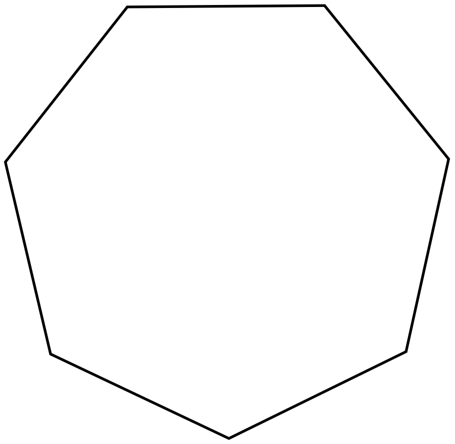
Road *B*

Road *C*



Road *D*

Road *E*

Number Line Tightrope

Brief Description:

This number line provides opportunities to explore families of numbers, positive and negative integers, patterns, and much more. Activities at this exhibit include demonstrating integer arithmetic, creating new number families, identifying additional values for the represented number families, and deriving the formulas that will yield the different number patterns displayed on the Number Line Tightrope.

Objectives:

These activities aim to expose students to various number families and their properties by observing patterns.

Links to Websites:

http://mathmidway.org/Training/numberline.php

http://mathmidway.org/Training/pdf/numberlineTourGuide.pdf

http://mathmidway.org/math-midway-puzzles-century1.php

http://en.wikipedia.org/wiki/Triangular_number

Vocabulary:

Cake Numbers
Counting Numbers
Factor
Fibonacci Numbers
Integers
Perfect Numbers
Powers of Two
Squares
Triangular Numbers
Zero
Constructible Polygon Numbers
Cubes
Factorials
Highly Composite Numbers
Pentagonal Numbers
Pizza Numbers
Primes
Tetrahedral Numbers
Whole numbers

Before:

- (*Level 1,2*) Begin by demonstrating that individuals share some human properties and some are unique to individuals. Ask all students with brown hair to stand in one area and all students with blue eyes to stand in another. Those students with both brown hair and blue eyes must share common space. Illustrate the outcome with a Venn diagram.

  Relate the idea of shared and unique properties to the number families, for example comparing the prime numbers to the even numbers.

  Define level-appropriate number families from the number line with the symbols.

  Create groups of students and ask them to come up with the properties that each group of numbers shares.

- (*Level 2, 3*) Learn about Triangular Numbers

Look at how bowling pins are arranged—in a triangle.

Provide dot paper for students to draw equilateral triangles and discover a geometric explanation of triangular numbers. http://en.wikipedia.org/wiki/Triangular_number

Make a chart of the triangular numbers according to the definition. (A number you get by adding up 1, then 1+2, then 1+2+3, then 1+2+3+4, etc.) Can you discover a pattern to find a large triangular number without having to do many additions? As triangular numbers can be displayed in a geometric display, how can square numbers be displayed geometrically?

- (Level 3) Derive a formula for the triangular numbers by studying the chart or by looking at the sum of two consecutive triangular numbers. [$T_n = \frac{n(n+1)}{2}$]

- (Level 1, 2, 3) Learn about Fibonacci Numbers in Nature: http://britton.disted.camosun.bc.ca/fibslide/jbfibslide.htm

- (Levels 2, 3) Activities for discovering many number families can be found on http://en.wikipedia.org/wiki/Main_Page using the search words: triangular numbers, factorials, perfect numbers, Lazy Caterer's Sequence (for pizza numbers) and highly composite numbers.

- (Level 3) Compass work to build 5-sided regular polygon (pentagonal numbers): http://www.kjmaclean.com/Geometry/PentConstruct.html

- Teachers should decide whether to show the training videos on the website http://mathmidway.org/Training/pdf/numberlineTourGuide.pdf before or after visiting the museum.

During:

- Investigations: (*Level 1, 2*) Gathered around the number line tightrope:
  Can you guess why different symbols are used for the same number or for different numbers?
  Which number families do you recognize?
  Of the number families tagged, which family has the greatest number of tags? Least?
  What familiar numbers do not appear on this model of the number line?

  Divide class into groups, with each group getting a number family to investigate. Worksheets should require defining the type of number, making a list of the numbers up to 100, and any interesting facts that the group finds.
  What symbol does the family of numbers have?

  After the investigation, groups can share their findings.
  Why do some numbers have more than one tag?
  What properties are shared? Unique?
  Can the students illustrate the shared and unique properties with a Venn diagram?

- (*Level 2, 3*) Find the numbers that are not integers. This provides an introduction to irrational numbers and infinity. Teacher should see brochure http://mathmidway.org/Training/pdf/numberlineTourGuide.pdf.

- (*Level 2*) Choose any tag on the number line. Good choices are the Fibonacci numbers, primes, squares, cubes, triangular numbers, or pizza numbers. Find all of the numbers that have that tag and make a chart. If the number line kept going, what would be the next three numbers to have that tag?

- (*Level 3*) Figure out the ratio of the two periods of the motors using a stopwatch to time the periods and note different intervals.

After:

- (*Level 1*) Explore the Sieve of Eratosthenes for finding prime numbers by downloading a worksheet from the following website:
  http://www.worksheetworks.com/math/numbers/sieve-of-eratosthenes.html

- (*Level 1*) There were no fractions on the number line. Choose some you consider appropriate, perhaps $\frac{1}{2}, \frac{5}{8}, \frac{4}{3}, \frac{15}{6}$, etc., and ask students to make a number line and place these fractions on it.

- (*Level 2, 3*) Create a new family of numbers for the number line.
  Ask students to think about what numbers would be included and what formula could be used to define them.
  Have students design a tag that defines the family of numbers.
  Make a number line for your classroom with the families you created. Make it go past 100.
  (For example: Even or odd, multiples of 7, powers of 3)

- (*Level 2*) Pick three categories for Venn Diagrams (ex. prime, triangular, fibonnacci), restricting the scale, and have students construct the diagrams.

- (*Level 2, 3*) To discover the triangular numbers in a physical context, try this handshake activity suggested by Elena Weinstein (NYC Lab School) and Aaron Orzech (The Urban Academy of Government and Law). The plan is referenced on momath.org, from a lesson plan written for Math For America, titled "Math Midway Field Trip Number Theory (advanced level).

Resources:

<u>What's Next? A Pattern Discovery Approach to Problem Solving</u> by Wilbert and Elaine Reimer (published by AIMS Educational Foundation, 1996) Contains life-related problems which address concepts from every area of mathematics and blackline masters to download.

Organ Function Grinder

Brief Description:

At this exhibit, students can explore the concept of functions in a fun musical context. Students can investigate many aspects of this topic, including estimating outcomes, figuring out which input(s) and functions will yield a desired outcome, composing functions, inverting functions, and graphing or charting the outcome of several inputs under the same function.

Objectives:

Students will understand that a function is a relationship between values where each input value determines exactly one output value. Students will conclude that there are multiple ways of arriving at a value. They will examine inverse relationships between values and discover the associative and commutative properties of addition and multiplication. For more advanced levels, students will connect musical composition with transformation geometry.

Links to Websites:

http://mathmidway.org/Training/organ.php

http://www.classicsforkids.com/terms/

http://score.kings.k12.ca.us/lessons/functions.htm

http://www.regentsprep.org/Regents/math/algtrig/ATP5/Lfunction.htm

Vocabulary:

Associative Property	Commutative Property
Distributive Property	Double
Function	Halve
Identity Element	Input
Inverse operation	Invert
Order of Operations	Output
Reciprocal	Square
Square Root	Undefined

Before:

- (*Level 1, 2, 3*) Teacher should explain the 8 available functions (add 3, subtract 3, square, square root, double, halve, invert, leave alone) or any of the functions appropriate to the group.

  After talking about function composition, practice composing functions. Group students into teams and give each team an output number. If your students are not familiar with function notation, you can make function machine diagrams (i.e. operations on the organ grinder) out of paper and give one to each student. Students will hold their diagrams and arrange themselves to reach goal outputs using an input of their choice. The first team to reach all goal outputs wins.

- (*Levels 1, 2*) Sometimes the order of the function matters! Discover when it does with the following examples:

Using the functions "*add 2*," "*multiply by 3*," and "*subtract 7*," determine an order in which they should be performed to get the required output from the given input.
input: 2 output:1 (ans. *multiply by 3, add 2, subtract 7* or *multiply by 3, subtract 7, add 2)*)
input: 7 output: 6 (ans. *subtract 7, add 2, multiply by 3* or *add 2, subtract 7, multiply by 3)*)
input: 5 output: 10 (ans. *multiply by 3, subtract 7, add 2* or *multiply by 3, add 2, subtract 7*)

Can you identify the operations when order doesn't matter?
Are these the only solutions?

Using the functions "*add 2*," "*multiply by 2*," and "*square*," determine the order they should be performed to get the output from the input.

input: 1 output:36 (ans. *add 2, multiply by 2, square,*)
input: 1 output: 16 (ans. *multiply by 2, add 2, square*)
input: 1 output: 18 (ans. *add 2, square, multiply by 2*)
input: 1 output: 4 (ans. *square, multiply by 2,add 2)*

During:

- (*Level 1, 2, 3*) Each student in a group of four is assigned one of the Function Grinder's operations. As a relay race a number is given to the first person in the group who performs the operation and writes the answer down to pass to the next person. The final result is given to a student to check by using the Organ Function Grinder. Students can rearrange themselves to get different results.

  Select 8 students to represent each of the functions on the Organ Function Grinder and have them stand. Write the function on a card that each student will hold. Docent selects an input number ticket and sets the knobs without letting the students see. Docent shares the result. Students must work to arrange three of the standing students in inverse order to produce the input number on the ticket. [Some students may catch on that each function has a distinctive sound.]

  Use the attached sheets of goal values. Give a sheet to each group of students, and they can try to find the input values and functions for as many of the outputs as they can. Students can select a few of their functions to test on the Organ Function Grinder.

- (*Level 3*) Investigations: Bobson Wong, Mathematics Teacher at Bayside High School in New York, is a Master Teacher with Math For America. He has submitted the attached plans and worksheets to the Museum of Mathematics to relate mathematical transformations to musical transformations. The plan is referenced on momath.org, from a lesson plan written for Math For America, titled "Field Trip on Functions."

- (*Level 2*) Use the lesson plan, designed by the Math Midway, for grades 4-9, accessible at momath.org.

After:

- (*Level 1, 2, 3*) Using the worksheet "Organ Function Grinder Activity: Reach the Goal!" study the ordering of functions and its effect on output values.

  Separate the class into groups or have students work individually.
  Students select 10 input values and then three functions.
  Students rearrange functions to examine the number of possible outputs.
  Pose the question: *"Whose inputs and function choices get the most outputs?*".

- (*Level 2, 3*) Create a worksheet that contains the following:
  What is a function?
  Why are the eight operations defined as functions?
  Create your own composition of functions using given inputs and determine if the composition is commutative.

Organ Function Grinder Activity: Reach the Goal!

Part I: Find the input and settings for the knobs that will give you the goal output. For the first chart, 2 of the knobs must be set to "leave alone."

Goal Output	Input	Knob 1	Knob 2	Knob 3
100				
20				
7.5				
49				
8				
0.5				
19				
78				
30				
1/9				

Part II: Using two functions and one dial set to "leave alone:"

Goal Output	Input	Knob 1	Knob 2	Knob 3
1/20				
11				
222				
17				
¼				
400				
-2.5				
-2				
84				
25				

Part III: Using three functions (no dials set to "leave alone"):

Goal Output	Input	Knob 1	Knob 2	Knob 3
403				
3 ¼				
124				
4/49				
17.5				
-10				
¼				
27				
3 1/9				
9997				

Answers (note there may be other possible answers)

Part I: Find the input and settings for the knobs that will give you the goal output. For the first chart, 2 of the knobs must be set to "leave alone."

Goal Output	Input	Knob 1	Knob 2	Knob 3
100	10	Square	leave alone	leave alone
20	10	Double	leave alone	leave alone
7.5	15	Halve	leave alone	leave alone
49	7	Square	leave alone	leave alone
8	64	square root	leave alone	leave alone
0.5	1	Halve	leave alone	leave alone
19	16	add three	leave alone	leave alone
78	81	subtract three	leave alone	leave alone
30	15	Double	leave alone	leave alone
1/9	9	Invert	leave alone	leave alone

Part II: Using two functions and one dial set to "leave alone:"

Goal Output	Input	Knob 1	Knob 2	Knob 3
1/20	10	Double	invert	leave alone
11	4	Double	add three	leave alone
222	15	Square	subtract three	leave alone
17	10	Double	subtract three	leave alone
¼	2	Invert	square	leave alone
400	10	Double	square	leave alone
-2.5	-8	add three	halve	leave alone
-2	2	Halve	subtract three	leave alone
84	9	Square	add three	leave alone
25	2	add three	square	leave alone

Part III: Using three functions (no dials set to "leave alone"):

Goal Output	Input	Knob 1	Knob 2	Knob 3
403	10	Double	square	add three
3 ¼	2	Invert	square	add three
124	8	add three	square	add three
4/49	7	Halve	invert	square
17.5	16	Double	add three	halve
-10	1	subtract three	subtract three	double
¼	4	Halve	invert	square
27	9	add three	double	add three
3 1/9	3	Invert	square	add three
9997	10	Square	square	subtract three

Mathematical Monkey Mat

Basic Description:
This whimsical mat that is the setting for the Polyhedral Puzzle Plaza provides a large-scale view of symmetry and tessellation.

Objectives:
At this exhibit, students can identify the points of symmetry in the mat and learn about different types of symmetry.

Links to Websites:
http://mathmidway.org/Training/monkey.php

http://www.learner.org/courses/mathilluminated/interactives/index.php#symm

http://www.teachersnetwork.org/dcs/math/symmetry/

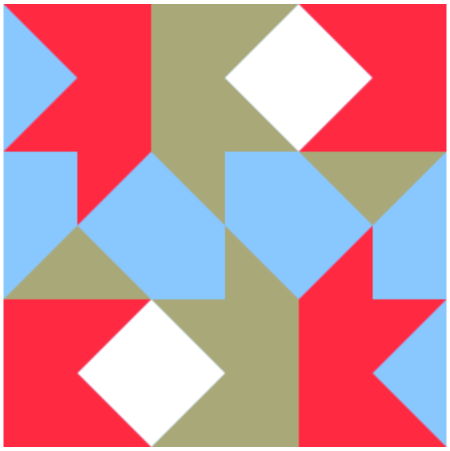
http://www.tessellations.org/diy-basic1.htm

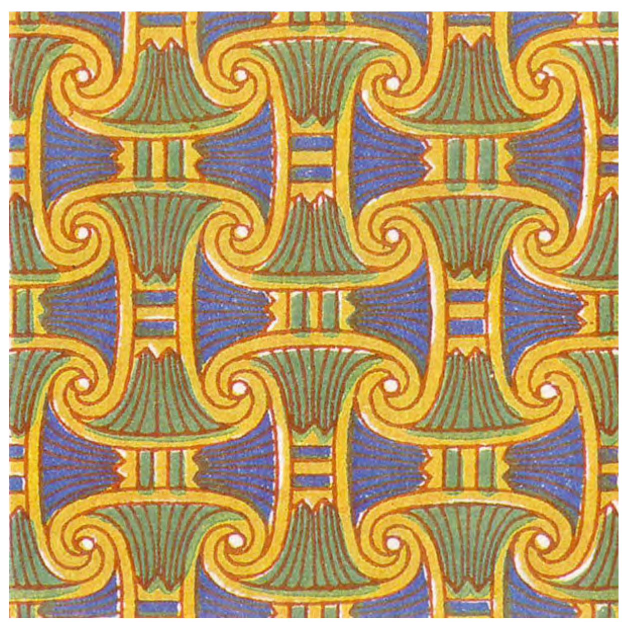
Vocabulary:
Axis of symmetry
Hexagon
Point symmetry
Rhombus
Rotational symmetry
Tessellation
Transformation
Dilation
Line symmetry
Reflection
Rotation
Six-fold symmetry
Three-fold symmetry
Two-fold symmetry

Before:
- (*Level 1, 2, 3*) Visit the Math Midway's training video for the Mathematical Monkey Mat: http://mathmidway.org/Training/monkey.php

- (*Level 1, 2, 3*) Learn or review the different types of symmetry by linking to: http://www.learner.org/courses/mathilluminated/interactives/index.php#symm and http://www.teachersnetwork.org/dcs/math/symmetry/

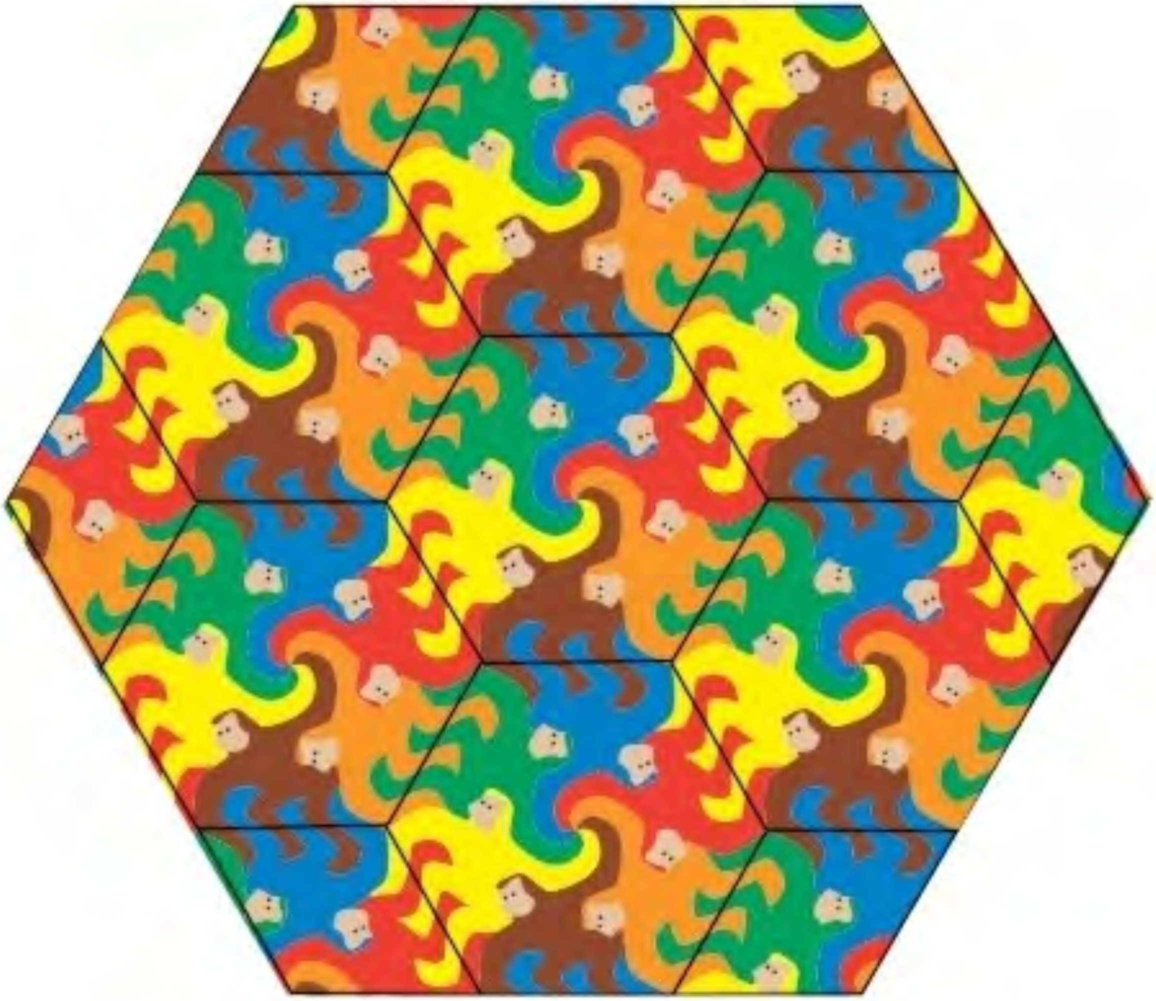
- (*Levels 2, 3*) Study other images to identify points of symmetry. Search the room, the internet and/or use the figures on the next page to locate symmetries.
  What type of symmetry do these objects have?
  If it (they) exist, can you draw the line(s) of symmetry?
  Locate the points of symmetry?
  Determine the angle through which the object must be rotated in order to view the identical image?





During:

- (*Level 1, 2, 3*) Look at the monkey mat and identify the points where the rotational symmetry is 2-fold, 3-fold, and 6-fold.



Answers:
2-fold symmetry point is where the ankles of the shorter legs touch,
3-fold symmetry point is where monkey elbows join,
6-fold symmetry point is where all the upturned arms meet

After:

- (*Level 1, 2, 3*) Create your own tessellation pattern on paper. A great interactive online tutorial is available here: http://www.tessellations.org/diy-basic1.htm

- (*Level 2, 3*) Use Geometer's Sketchpad to create your own tessellations.

- (*Level 2, 3*) Investigations: Examine MC Escher's work.

Polyhedral Puzzle Plaza

Basic Description:

There are three different three-dimensional puzzles found here: a tetraxis, the soma cube and the burr puzzle. Children work on the mathematical monkey mat to assemble the puzzles.

Objectives:

Opportunities to investigate three-dimensional geometry abound at this exhibit. Students can solve large geometrical puzzles, build different solids, explore geometrical properties, and relate surface area with volume.

Links to Websites:

http://mathmidway.org/Training/polyhedral.php

http://www.geom.uiuc.edu/docs/forum/ElPuz/

http://www.fam-bundgaard.dk/SOMA/FIGURES/FIGURES.HTM

Vocabulary:

Cube	Polyhedron
Soma cube	Surface area
Tetraxis	Volume

Before:

(All of the following activities can be accommodated for all levels.)

- Familiarize the class with geometric puzzles such as Soma cubes and tetraxis.

- Play Pentaminoes or Tetris on Smartboard. Search for web or physical versions of them. (Nearly every video game console contains a version of Tetris.)

- Explore The Elephant Tetrahedron Puzzle at: http://www.geom.uiuc.edu/docs/forum/ElPuz/)

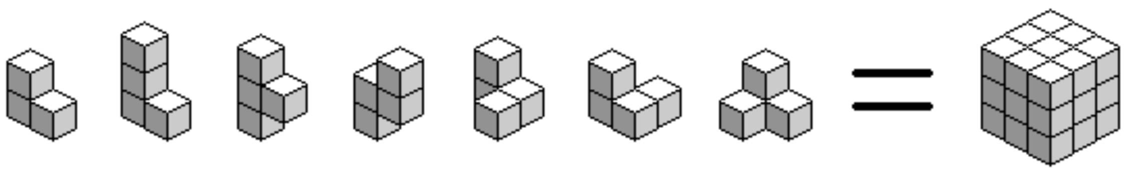
During:

- Play the Soma cube race:
  Assign two groups to the soma cube. One group will attempt to put the soma cube together as the other group waits.
  Record the times.
  See if they can beat their times or build other shapes.

- Play the Tetraxis race:
  Two groups will study a tetraxis.
  The groups will take them apart and attempt to put the tetraxis pieces back together.
  Record times.
  See if they can beat their times.

After:

- You can purchase small soma puzzle sets from http://thinkfun.com/ called "Block by Block." You can also make them in the classroom by gluing together small cubes. You can use sugar cubes from the grocery store or wooden/foam cubes available from http://www.enasco.com/. 27 cubes will make up the 7 pieces shown below. Be careful to build each piece to match the picture, especially the two twisted ones that have different "handedness."



- Find puzzles to solve here: http://www.fam-bundgaard.dk/SOMA/FIGURES/FIGURES.HTM

Traveling Carnival

Brief Description:
There are twelve poles, each marked with the name of a hypothetical city. Visitors try to plan the shortest route that visits all twelve cities in a loop, ending where it started. Using the surveyors' wheels, they then walk the route and measure the distance. The shortest route is posted and subsequent participants try to find a shorter route.

Objectives:
Students can learn about the basics of networks and optimization at this exhibit based on a very practical real-world problem. Activities include identifying the route that appears optimal, measuring the total distance traveled, and estimating distances to assess the reasonableness of their result.

Links to Websites:
http://mathmidway.org/Training/carnival.php

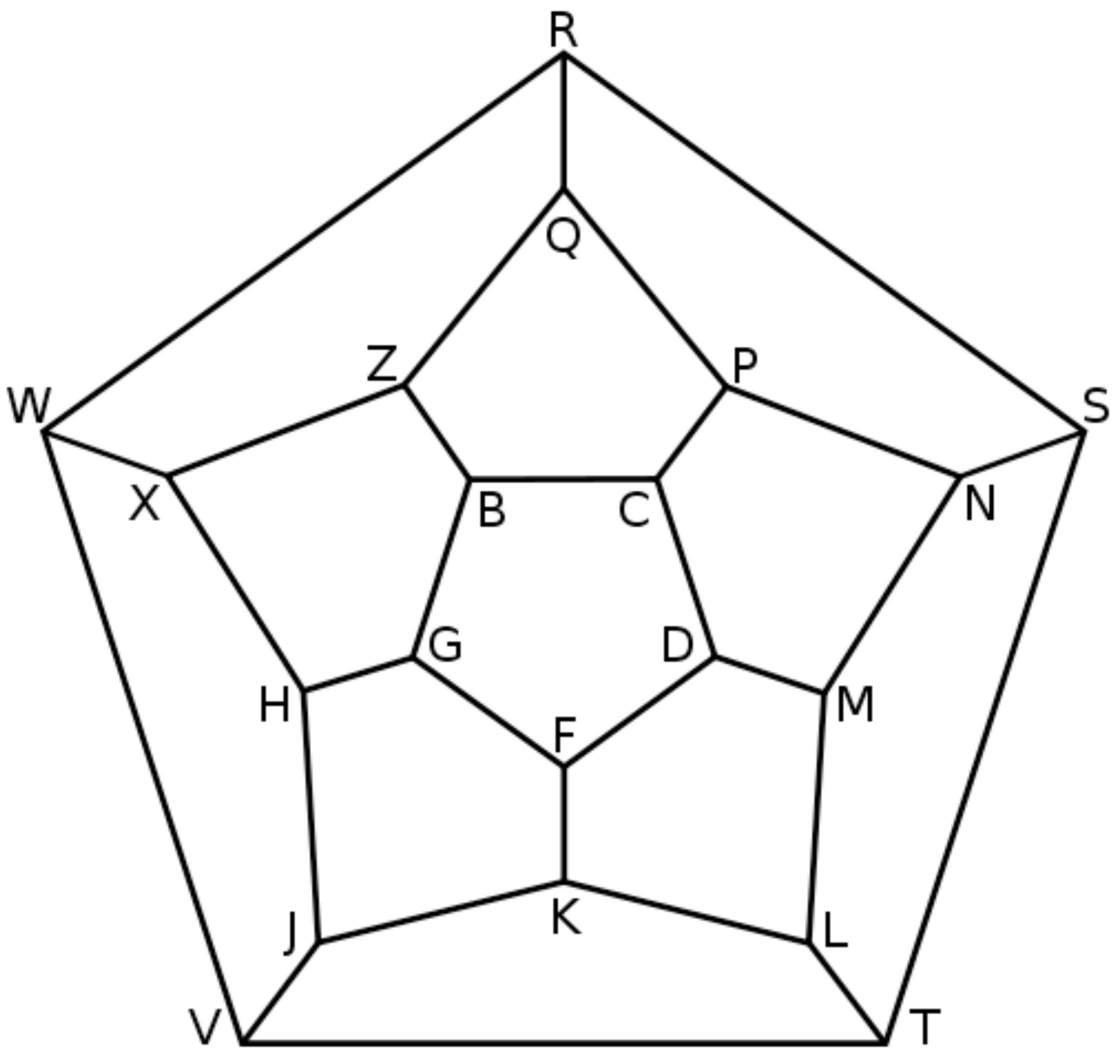
http://maps.google.com

Vocabulary:
Circuit	Distance between two points
Dodecahedron	Euler's circuit
Graph	Hamiltonian path
Path	Platonic solid
Vertex	

Before:
- (*Level 1, 2, 3*) Discuss forming your own path.

- (*Level 2, 3*) A great topic to explore before seeing the Traveling Carnival is the ***Hamiltonian circuit***. This is a path that visits every vertex of a graph once and only once. The end point must be the same as the starting point in order to form a circuit. The route students take at the Traveling Carnival exhibit will also involve visiting every location in space exactly once and then returning to the start point.

- A fun Hamiltonian puzzle known as the Icosian game can be found on the following sheet. Icosian Game: Find a path around the shape so that you visit every vertex (the lettered points) exactly once and end at the starting point.



During:

- (*Level 2, 3*) Have each student or small groups of students plan a route and walk it. Record the routes and distances to examine whose route is the shortest. *Does the starting location affect the efficiency of the route?*

After:

- (*Level 1, 2, 3*)Using the map on the following page, visit each city on the list below. Make sure to return to the starting city. Calculate the total mileage by using the ruler on the map. Measure straight as if you were flying directly between the cities. Google Maps, http://maps.google.com, can be used to calculate the total mileage. Use "Get directions" function to find the mileage between the cities if you drive. Use the first suggested route when getting the mileage from Google Maps.
  Here are the cities you need to visit:
  Albuquerque, NM
  Atlanta, GA
  Denver, CO
  Houston, TX
  Philadelphia, PA
  Seattle, WA
  Wichita, KS

nationalatlas.gov™ Where We Are

GENERAL REFERENCE

U.S. Department of the Interior
U.S. Geological Survey

The National Atlas of the United States of America®

genref1.pdf INTERIOR-GEOLOGICAL SURVEY, RESTON, VIRGINIA-2003

A-Maze-ing Math

Brief Description:
This exhibit consists of maze(s) where one has to go from start to goal by following the path, but observing some additional rules in the journey.

Objectives:
Spatial reasoning and graph theory combine in this maze with a challenging twist. Activities include trying to solve the maze with multiple paths and finding the shortest route as well as determining whether the maze has any unnecessary segments for the solution.

Links to Websites:
http://mathmidway.org/Training/amazeing.php

http://www.falstad.com/maze/

Vocabulary:
Left turn
Right turn
Maze

Before:
- (*Levels 1, 2, 3*) The A-Maze-ing Math exhibit is about navigating a maze with certain rules. Here's an online maze that is surprisingly difficult to navigate, because it's in 3D. There are many levels of difficulty to choose from. The applet generates different mazes each time, so students will not encounter the same mazes. http://www.falstad.com/maze/
- (Have students draw an aerial view of the 3D maze they're trying to solve. Drawing the aerial view may help to solve the mazes.)

During:
- (*Levels 1, 2, 3*) Solve the maze. Discuss the strategies that were used. Working backwards may help some students (solve the maze from finish to start, making no right turns). Ask them to think of a path that turns in one direction (ans. spiral). This may also help them find the solution.

After:
- (*Levels 2, 3*) Design a maze that can be solved without using any left turns. Students may use graph paper or drawing software.

  Some additional mazes: http://www.clickmazes.com/

Miles of Tiles

Brief Description:
This exhibit consists of a magnetic wall and different polygons and shapes which the students use to create patterns and tessellations.

Objectives:
While students express creativity with the patterns they make at this open-ended exhibit, they learn about symmetry, tessellation, and two-dimensional geometry. Activities include making tessellations of one or more polygons, learning about Penrose tilings, exploring geometric transformations, and investigating the properties of the different polygons.

Links to Websites:
http://mathmidway.org/Training/tiles.php

http://www.tessellations.org

http://www.tessellations.org/diy-basic1.htm

http://www.cgl.uwaterloo.ca/~csk/software/penrose/

http://stephencollins.net/penrose

Vocabulary:
Area
Exterior angle
Iteration
Penrose tilings
Rectangle
Square
Transformation
Hexagon
Interior angle
Parallelogram
Polygon
Rhombus
Tessellation
Triangle

Before:
- (*Levels 1, 2, 3*) Discuss what a tessellation is and why only certain regular polygons will tile the plane. http://www.tessellations.org

- (*Levels 1, 2*) Expose students to different shapes with pattern blocks.

During:
- (*Level 1, 2*) Activity: "Adding to One"
  Show example of how 4 square magnets placed together can make a larger square. (Each of the smaller squares is 1/4 of the large square.)

  Take 6 triangular magnets and place them around a point.
  What fraction is each triangle in the larger polygon? (ans. 1/6)

  Ask students to think of another available magnetic polygon that can surround a point.
  What fractional part of the "whole" is each polygon?

- (*Level 2, 3*) Activity "Adding to 360"
  Create student groups and have them explore the different combinations of polygons that can surround a point, and ask them to discuss the fraction of the whole each type of polygon occupies. The answer will depend upon whether they count the number of polygons, the number of degrees or the area.

  Discuss the interior angles of the polygons.

  In groups, students explore the different combinations of polygons that can surround a point, and ask them to record the angle measurements to make sure they add up to 360°.

- (*Level 1, 2, 3*) Pattern-making activity
  Show an example of a tessellation.
  Send students to either side of the tile wall to extend the patterns.
  Discuss the types of symmetry and strategies for extending the pattern.

After:

- (*Levels 1, 2*) Investigate Penrose tilings at: http://stephencollins.net/penrose and generate a Penrose tiling using an applet at: http://www.cgl.uwaterloo.ca/~csk/software/penrose/

- (*Levels 1, 2, 3*) Make tessellating shapes in your classroom with construction paper. Use this website as a guide to help create tessellations. http://www.tessellations.org/diy-basic1.htm

- (*Levels 2, 3*) Create a tessellation on graph paper and find the area of the creation by calculating the area of one iteration of the pattern.

Plant the Daisy

Brief Description:

The larger "daisy" in this exhibit is attached to a 10-foot tall pole. The participant will try to place this pole over a small post sticking up from the center of a weighted pot. The task seems easy at first, but the pole is surprisingly difficult to control when held in one hand at the nodal point. Small children can use the miniature version.

Objectives:

Students will learn about the different parts of a wave in a very unique way at this challenging exhibit. Plant the Daisy provides a great jumping-off point to connect math to real world situations, such as how musical instruments work and the types of problems architects may encounter when building structures.

Links to Websites:

http://en.wikipedia.org/wiki/Normal_mode

http://mathmidway.org/Training/daisy.php

http://hyperphysics.phy-astr.gsu.edu/hbase/Sound/stawav.html

http://videos.howstuffworks.com/tic/29833-understanding-tacoma-narrows-bridge-video.htm

Vocabulary:

Amplitude	Antinode
Dampening	Frequency
Fundamental node	Harmonic node
Node	Normal mode
Oscillating system	Phase
Sinusoid	Standing wave
vibration wave theory	

Before:

- *(Level 3)* A vibrating object (such as a guitar string) has different modes of vibration. These different modes are different ways in which the vibration happens. The fundamental mode of vibration occurs when the most vibration happens in the middle of the object and the ends don't move at all.

  You can demonstrate this with a slinky in your classroom. It's easiest to do if you wave the slinky along the floor and have students around it. Stretch the slinky along the floor. A student at one end will hold the slinky still. The student at the other end starts waving the slinky until the fundamental mode is achieved. This pattern will show the largest amount of movement (an antinode) in the center of the slinky, and the ends are nodes, where the amount of movement is smallest.

  Once you've established the fundamental mode of slinky vibration, the student who is waving the slinky can begin waving faster to achieve the 2nd mode of vibration. He or she will have to wave the slinky twice as fast to achieve this mode. When the 2nd mode is achieved, you will be able to see a node in the center of the slinky and antinodes between the center and the students. Continue waving faster and faster to see how many nodes and antinodes you can achieve.

See a video of how standing waves look with Slinkys:
http://hyperphysics.phy-astr.gsu.edu/hbase/Sound/stawav.html

During:

- (*Level 1, 2, 3*) Hold the daisy at a variety of places along the stem. Observe the amount of wiggle in the stem.

- (*Level 3*) Try to plant the daisy! See if you can estimate the frequency of the wave by counting the wobbles.

After:

- (*Level 3*) Watch a video of the Tacoma Narrows bridge to see the effect of vibration on architectural structures. http://videos.howstuffworks.com/tic/29833-understanding-tacoma-narrows-bridge-video.htm

Math Unleashed

Brief Description:
There are three separate puzzles here, at three neighboring stations. For the first two puzzles, the docent sets up the leash, following the diagram on the sign board. The participant then tries to solve the puzzle, figuring out which post can be lifted up to let the dog free. (The dogs are stuffed animals, attached to actual leashes.) The docent can ask the participant, "Which post should I lift?" and once the participant chooses, the docent lifts the post while the participant tries to pull the dog free. If they chose the correct post to lift, the leash will untangle itself from the remaining post when the dog is pulled.
The third puzzle is much harder than the other two. The participant designs a way to wrap the leashes around both posts, so that lifting either one of the posts will allow the dog to be pulled free.

Objectives:
This exhibit provides a surprising example of operations and inverses. It also reinforces spatial reasoning for students of all ages. Activities at this exhibit include solving the puzzles, creating new puzzles, and learning how to express problems of this type and their solutions using mathematical notation.

Links to Websites:
http://mathmidway.org/Training/unleashed.php

http://library.thinkquest.org/12295/data/Knots/Articles/Knots_I_1.html

Vocabulary:
Group theory
Isotopy
Operation
Inverse
Knot theory

Before:
- (*Level 1, 2, 3*) Figure out a way to pick up both ends of a string and tie a half-knot without letting go of either end and without having your hand or arm inside the knot.

- (*Level 2, 3*) Many mathematicians study the relationships that exist in diagrams of tangled-up string, like the puzzles in the Math Unleashed exhibit. Students can get a sense of how to untangle something that seems very tangled up in a fun and kinesthetic way by playing the Human Knot game.

  Instructions for Human Knot game:
  Divide the class into groups of 6-10 people.
  Each group forms a tight circle, standing and facing each other.
  Everyone extends their hands into the circle and grasps hands with other members of the group.
  Everyone needs to make sure that the two hands they are holding do not belong to the same person.

The groups' goal: untie the knot that results after holding hands. Members of the group physically climb over/under/through each other's arms to untie the knot. Note: It is possible for a knot to be unsolvable or end in two separate circles.

- (*Level 3*) Study the Reidemeister Moves at the website: http://library.thinkquest.org/12295/data/Knots/Articles/Knots_I_1.html
  Hand out pieces of string to give students a chance to reproduce the three types of moves.

During:

- (*Levels 2, 3*) Solve both puzzles and try the challenge for the third puzzle.

After:

- Watch knot doodling videos at http://vihart.com/doodling/.

- Have a student hold 6 strands of rope all the same length in one hand. By twisting the 6 strands together matching the ends of the rope will become difficult. Have a student select two ends from one side of the ends coming out of the student's hand and tie the ends together. Have a second student repeat the process with the remaining ends. Have a third student tie the remaining two ends in a knot. Now choose three students to repeat the activity with the 6 ends on the other side of the student's hand. Ask the class the following three questions:
  What is the probability the result will be 1 loop containing all 6 strands? (8/15)
  What is the probability the result will be 2 loops one of 4 strands and one with two strands?
  What is the probability the result will be 3 loops each contain ing two strands? (1/15)

Amazing Acrobats

Brief Description:
This exhibit consists of model "acrobats" that are screwed together, initially to form twelve cycles of five connected figures, which eventually can be connected to form a dodecahedron.

Objectives:
Here is an opportunity to explore symmetry and three-dimensional geometry, through the construction of a beautiful mathematically based sculpture. Students can choose color patterns for the sculpture that highlight the different symmetries of the completed shape.

Links to Websites:
http://mathmidway.org/Training/acrobats.php

http://www.enchantedlearning.com/math/geometry/solids/Dodecahedrontemp.shtml

http://www.ii.uib.no/~arntzen/kalender/

http://web.eecs.utk.edu/~plank/plank/pics/origami/penultimate/pentagon.html

http://www.cs.utk.edu/~plank/plank/pics/origami/penultimate/dodecahedron.html

Vocabulary:
Dodecahedron
Face
Origami
Rhombus
Edge
Hexagon
Pentagon
Vertex

Before:
- (*Level 1, 2, 3*) Make a model of a dodecahedron, which will introduce the shape of the completed exhibit sculpture. Use template at this website: http://www.enchantedlearning.com/math/geometry/solids/Dodecahedrontemp.shtml

- (*Level 1, 2, 3*) Another website that allows you to print out a dodecahedron calendar can be found at: http://www.ii.uib.no/~arntzen/kalender/. It also allows the visitor to print out a rhombic dodecahedron calendar. Both sites give the folding directions and offer the visitor a variety of options relating to the year, language, and starting day.

  Count the faces, edges, and vertices.
  If the sculpture has 30 pieces in it, what must the pieces represent? [There are 12 faces, 20 vertices, and 30 edges, so each piece represents an edge.]

During:
- (*Level 1, 2, 3*) Build the sculpture as a group and try to incorporate a particular color pattern, for example, make each 5-acrobat "pentagon" out of a single color.

After:

- (*Levels 1, 2, 3*) Modular origami is an easy way to incorporate three-dimensional mathematical sculpture into your classroom. A great place to start is with a modular dodecahedron. This is made with 30 pieces of 4"x3" paper folded into "pentagon modules." The instructions for folding the pentagon module can be found here:
  http://web.eecs.utk.edu/~plank/plank/pics/origami/penultimate/pentagon.html

  The instructions for putting the modules together into dodecahedrons are here:
  http://www.cs.utk.edu/~plank/plank/pics/origami/penultimate/dodecahedron.html

  Unfolding origami creations can reveal many geometric shapes in the crease patterns.

Resources:

Polyhedron Origami for Beginners by Miyuki Kawamura
Unit Origami: Multidimensional Transformations by Tomoko Fuse

Ring of Fire

Brief Description:
A laser plane creates a two-dimensional cross-section of a three-dimensional object when the object is placed in the ring of fire.

Objectives:
This exhibit helps students to connect two- and three-dimensional geometry by illuminating the polygons hidden within a polyhedron's cross-section. Students can predict and describe the different and often surprising shapes they can find within various translucent solid models.

Links to Websites:
http://mathmidway.org/Training/fire.php

http://www.clausentech.com/lchs/dclausen/algebra2/conic_sections.htm

http://www.learner.org/courses/learningmath/geometry/session9/part_c/index.html

http://www.mhhe.com/math/ltbmath/applets/ch9/

http://demonstrations.wolfram.com/CrossSectionsOfRegularPolyhedra/

Vocabulary:
Circle	Cone
Conic section cube	Cross section
Cylinder	Decagon
Ellipse	Hexagon
Hyperbola	Octagon
Parabola	Parallelogram
Pentagon	Plane
Polygon	Rectangle
Square	Trapezoid
Triangle	

Before:
- (*Level 2, 3*) Review the website:
  http://www.clausentech.com/lchs/dclausen/algebra2/conic_sections.htm

  Give students a chance to reproduce the conic sections with a flashlight.

- (*Level 1, 2, 3*) Review polygons up to the decagon (as well as conic section shapes - ellipse, circle, parabola, hyperbola- for levels 2 and 3)

- (*Level 3*) Use "The Graphing Calculator" or other graphing software to view both the 3-D and 2-D versions of the conic sections simultaneously.

During:
- (*Level 1, 2, 3*) "Predict and find" activity

Take a 3-D solid and put it through the Ring of Fire. Explain how the lasers act like a knife and slice through the solid to show a 2-D cross section.
Ask students what other 2-D cross sections they might find.
Review all the 3-D solids and possible 2D cross sections.
Group students and hand out a 3-D solid. Ask them to predict what 2-D cross sections they may find.
Take groups one at a time to the Ring of Fire to test out the 3-D solid and compare the actual cross sections to their predictions.
Discuss the results.

After:

- (*Levels 1, 2, 3*) Make 3D solids out of clay and let students experiment with different cuts to produce cross sections. Then view the interactive website: http://www.learner.org/courses/learningmath/geometry/session9/part_c/index.html

- (*Levels 1, 2, 3*) Visit applet: http://www.mhhe.com/math/ltbmath/applets/ch9/ to create cross-sections within a cube by selecting edges of the cube. http://www.tricounty.k12.ia.us/TeacherPages/Ward/GeomApplets/geo3apd/index.html

- (*Levels 2, 3*) Investigations: research Magnetic Resonance Imaging (MRI) and Computerized Axial Tomography (CT) Scan.

Roller Graphicoaster

Brief Description:
In this exhibit you adjust the shape of a track and roll a cart along it, timing how long it takes from start to finish.

Objectives:
Students explore slopes and functions with this exciting exhibit. Students can analyze their results to find the fastest possible path or to compare their average race times with the results of their classmates.

Links to Websites:
http://mathmidway.org/Training/graphicoaster.php

http://curvebank.calstatela.edu/index/index.htm

Vocabulary:
Brachistochrone	Catenary curve
Cosine curve	Cubic
Cycloid	Parabolic curve

Before:
- (Level 1, 2, 3) You can prepare to use the Roller Graphicoaster exhibit in the Math Midway by thinking about the question: What is the shape of the curve that will allow an object to slide from one point to another point in the least amount of time if the only force that is setting the object in motion is gravity?

  The following worksheet can accompany the classroom query:

Draw a curve that you think will allow an object to get from point A to point B in the least amount of time.

● A

● B

Why do you think this particular curve will be so fast?

Draw a curve that you think will allow an object to get from point A to point B, but as slowly as possible.

● A

● B

Why do you think this particular curve will be so slow?

Draw a curve that you think would allow the object to slide from point A to point B slower than the first curve you drew but faster than the second.

● A

● B

Why do you think this curve would be between the first two?

During:

- (*Levels 1, 2, 3*) Try to figure out the fastest curve
  Each student will test out a design that they think will work best.
  Record the design, time, and student's name.
  Discuss the slope, velocity, and acceleration of the fastest and slowest curves.
  Review the curve designs available at the exhibit.
  Ask students to compare their curve designs with those.

After:

- (*Level 3*) The question that this exhibit is based on is called the "Brachistochrone Problem," which examines the curve that will allow an object to slide from one point to another in the least amount of time. Isaac Newton was challenged to answer this question in 1696, and he answered the question using the calculus of variations. While your students may not be ready for calculus yet, they can still think about this question.

- (*Level 3*) Visit the National Curve Bank:
  http://curvebank.calstatela.edu/index/index.htm Form groups of students to research different curves or curve families and have them present their work to the class.

Mirror Morph

Brief Description:
At this exhibit, students can investigate the reflective properties of mirrors by manipulating their surfaces.

Objectives:
After learning how light reflects off a surface, students can try to predict and describe the effect of different mirror distortions on their reflections.

Links to Websites:
http://mathmidway.org/Training/mirror.php

http://www.staff.olympia.org/external/classes/Staudenmeier/Student_Page/geometry/Miniature_Golf/Tutorial.html

http://online.math.uh.edu/MiddleSchool/Modules/Module_4_Geometry_Spatial/Activities/Hole_One/UHGeometryHoleinOneTeacherNotes.pdf

Vocabulary:
Angle of incidence	Angle of reflection
Concave	Congruence
Convex	Glide-reflection
Image	Isometry
Optics	Perpendicular bisector
Pre-image	Reflection
Refraction	Rotation
Transformation	Translation

Before:
- (*Level 1, 2, 3*) Explore angle of incidence and angle of reflection in real life (mirrors, miniature golf, billiards, Wii, etc.)

  http://www.staff.olympia.org/external/classes/Staudenmeier/Student_Page/geometry/Miniature_Golf/Tutorial.html

  http://online.math.uh.edu/MiddleSchool/Modules/Module_4_Geometry_Spatial/Activities/Hole_One/UHGeometryHoleinOneTeacherNotes.pdf

- (*Level 1, 2, 3*) Show how light is reflected off a smooth surface.
  Collect the following materials, mirror, two cardboard tubes, and a flashlight.
  Place the mirror on a flat surface.
  Hold one tube at an angle with the end touching the mirror.
  Ask another person to hold the second tube at the same angle. 5. Shine the flashlight into the tube.
  If the tubes are at the same angles, the light will bounce off the mirror and down to the end of the second tube.

During:

- (*Level 1, 2, 3*) Have students write down in a notebook their observation of the two mirrors. Discuss concave and convex curves. Ask students to describe the shape of the mirror and the distortion of their reflection. Include observations about how the image changes as they move closer and farther away from the mirror.
  Which mirror makes you look wider, narrower?

After:

Investigations: Discuss various shaped reflectors and their applications.

Coffee Cup Curves

Brief Description:
This exhibit consists of a circular table with a printed top, a bright spotlight, and flexible plastic mirrors. Students explore the curves of light that are created by the reflections of straight lines against a curved surface.

Objectives:
Students will attempt to solve the different shape challenges offered by the exhibit, or they can create their own interesting reflection patterns.

Links to Websites:
http://mathmidway.org/Training/coffee.php

http://www.math.admu.edu.ph/~jumela/EDSC320/Conic%20Sections.pdf

http://www.mathinscience.info/public/conic_sections/folding_conic_sections.pdf

http://www.mathcats.com/crafts/stringart.html

Vocabulary:

Caustic curves	Circle
Circumference	Conic sections
Directrix	Ellipse
Focus	Hyperbola
Nephroid	Parabola
Spirograph	Tangent

Before:
- (*Levels 2, 3)* Straight lines can be used to construct conic sections (parabola, ellipse, circle, hyperbola). The following websites provide instructions and explanations for the "how-to."

  http://www.math.admu.edu.ph/~jumela/EDSC320/Conic%20Sections.pdf

  http://www.mathinscience.info/public/conic_sections/folding_conic_sections.pdf

- (*Levels 1, 2*) The Coffee Cup Curves exhibit shows you how you can make curves out of straight lines. In the case of the exhibit, these straight lines are the rays of light emanating from the lamp above the exhibit. You can demonstrate this idea with your students using string art. You can make these pictures using paper and a straight edge, or if you have the materials, you can use colored thread, straight pins, and foam.
  A good website for the basic string art patterns is here:
  http://www.mathcats.com/crafts/stringart.html

- (*Level 3*) For a more advanced string art pattern, you can make a nephroid shape, which is one of the curves you will be able to see at the Coffee Cup Curves exhibit. (See below for instructions.)

The nephroid is the path followed by a point on the circumference of a circle of radius one unit as it rolls around the circumference of a circle of radius two units. It is formed here from its tangents by laying N points equally on a circle, and joining point n to the point (3n + N/2) mod N. The nephroid may be demonstrated using spirograph techniques, using either the toy or a computer application.

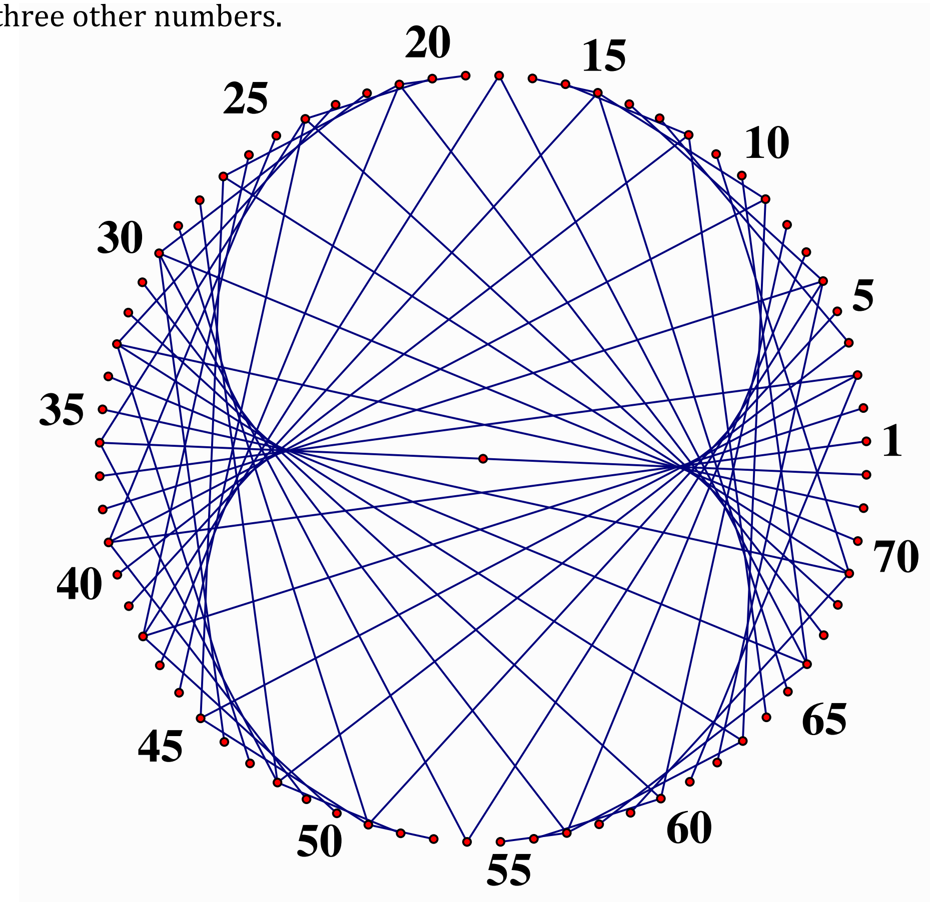
The nephroid may also be formed by rolling a circle of radius *r*, around the outside of a circle of radius 2*r*. *What curve does rolling a circle of radius r inside a circle of radius 2r form?*

Drawing a Nephroid

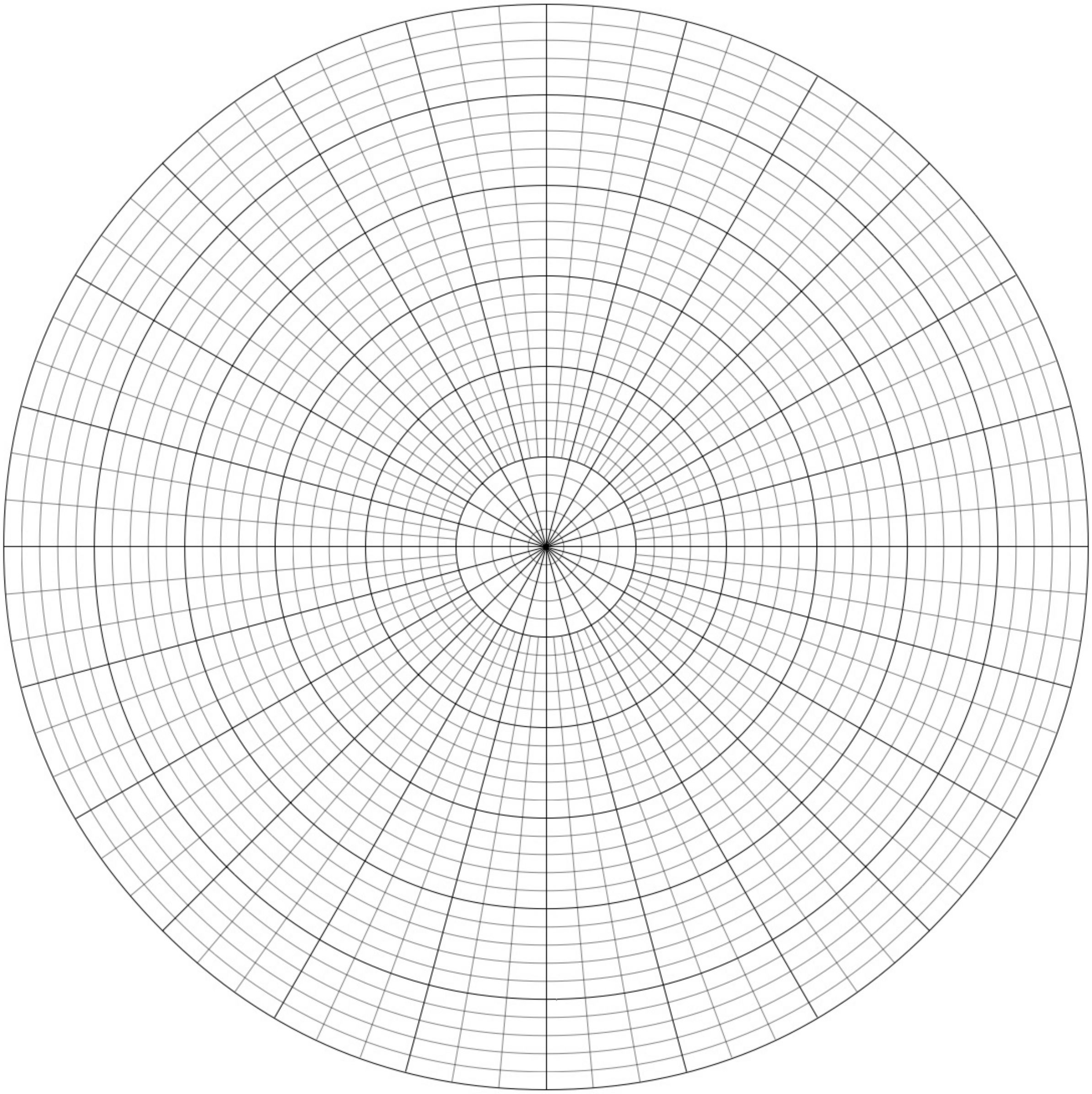
Draw a point at each location on the circle where a radius intersects with the outer circle. Number these points starting with 1. There should be 72 of these points.

To figure out which points to connect to make the nephroid shape, fill out this chart. The first several are done as an example. For each number, multiply it by three and then add 36. Put this number in the second column. The third column is found by dividing the number in the second column by 72, and taking the remainder. This can be done very nicely in a spreadsheet such as Excel. The second column may be omitted since you can use the formula: *=mod(3*A1+36,72)* in column B when column A are the integers from 1 to 72.

Connect the numbered dot with the number in the remainder column with a straight line. The numbers 12, 36, and 60 need to connect to the number 0, but 0 is not one of the numbered points. Just ignore these since each of these numbers will be connected to the following sets of numbers: 12 connects to 16, 40, and 64, 36 connects to 24, 48, and 72, and 60 connects to 8, 32, and 56. You might also notice that all numbers divisible by 3 connect to three other numbers.



Number (n)	$3n+36$ (b)	Remainder of b divided by 72	Number (n)	3n+36 (b)	Remainder of b divided by 72
1	39	39	37	147	3
2	42	42	38	150	6
3			39		
4			40		
5			41		
6			42		
7			43		
8			44		
9			45		
10			46		
11			47		
12	72	0	48		
13			49		
14			50		
15			51		
16			52		
17			53		
18	90	18	54		
19			55		
20			56		
21			57		
22			58		
23			59		
24			60		0
25			61		
26			62		
27			63		
28			64		
29			65		
30			66		
31			67		
32			68		
33			69		
34			70		
35			71		
36		0	72		



During:

- (*Levels 1, 2, 3*) Explore the activities suggested by the graphic on the Coffee Cup Curves table.

After:

- Look for examples of nephroids in the real world

Funny Face

Brief Description:
Participants sit in front of the camera and the machine takes 4 pictures in succession. The first picture is not altered. For each of the other three, the computer randomly picks one mathematical transformation from a built-in library, and shows the picture with that transformation. The user can adjust the slider on the screen to change the amount of distortion in the transformation, and see how that change affects the picture.

Objectives:
Students will see how a mathematical equation can be used to distort an image. They will learn more about graphs and see firsthand the effect of some unusual transformations.

Links to Websites:
http://mathmidway.org/Training/face.php

Vocabulary:
Abscissa	Cartesian coordinates
Ordinate	Parameter
Pixel	Polar coordinates
Reflection	Rotation
Scale	Translation

Before:
- (*Level 1, 2*) Give groups of students a piece of silly putty to roll out flat on the table. Draw a picture of something and transfer it onto the silly putty. Now stretch the silly putty to distort your drawing.
- (*Level 1, 2, 3*) Create rubber band secret messages by writing on a stretched rubber band and then letting it shrink.
- (*Level 1, 2, 3*) Draw shapes on a balloon. Stretch the balloon and discuss the distortion.
- (*Level 2, 3*) Activities like those listed above can also be done on a computer using any draw program (Illustrator, Intaglio, Concept Draw, etc.) or Photoshop. They all contain options to distort/transform an image.

During:
- (*Level 1, 2, 3*) Each student should take his/her photo and distort the image, as per instructions on the training video.
- (*Level 2, 3*) Record the formula for the distortion.
  How does the image change as a value (parameter) in the formula is increased or decreased?

After:

- (*Level 2, 3*) Distorting a polygon:

1. On a sheet of graph paper, mark the origin and draw the *x*- and *y*-axis.
2. Draw a polygon whose vertices are at points on the Cartesian plane. Use any polygon you like. Pentagons work very well for this exercise.
3. Label each of the vertices with a letter and write the coordinates that correspond to the letter elsewhere on the paper, for example: $A(1, 1)$, $B(1, 3)$, $C(2, 4)$, $D(3, 3)$, and $E(3, 1)$
4. Transform this shape similarly to how Funny Face distorted your photo. The first transformation is this $x' = x + y$ and $y' = x - y$. Figure out what each of the new coordinates for your shape would be. For the example shape listed above, they would be: $A'(2, 0)$, $B'(4, -2)$, $C'(6, -2)$, $D'(6, 0)$, and $E'(4, 2)$.
5. Plot the new points on the graph paper. Use a different color ink or pencil to draw the new shape. *How has the shape changed?* (Has it been scaled, reflected, rotated, and/or translated.)
6. Try another distortion formula. This time try $x'' = 2x + y$ and $y'' = x + 2y$. For this distortion on the original example coordinates, the new coordinates become: $A''(3,3)$, $B''(5,7)$, $C''(8,10)$, $D''(9,9)$, and $E''(7,5)$.
7. Plot the new shape and describe how it looks now.
8. Have students make up their own distortion equations and plot them on graph paper.

Investigations*: What happens when you take your original shape, apply the first distortion, and then apply the second distortion to the new coordinates instead of the original ones? How do you think the funny face exhibit was distorting the image coming from the webcam at the exhibit?*
[Answer: If you look at a computer screen closely, the image is made of pixels, which are tiny divisions of the entire image. Each of those pixels has coordinates, just like the vertices of the polygons from this activity.]

The Mysterious Harmonograph

Brief Description:
The harmonograph is a machine for creating mathematical drawings. Students begin by putting a piece of paper under the middle or one end of the bar and holding the paper down with a couple of magnets. The harmonograph is set in motion; once the bar has settled into a regular motion, pen(s) can be added to create drawings.

Objectives:
Students can explore fascinating wave motion in a two-dimensional plane with a pendulum that creates beautiful curved drawings similar to Lissajous figures. Several different variables can be manipulated to create unique drawings, and students can investigate the effect of changing each control.

Links to Websites:
http://mathmidway.org/Training/harmonograph.php

http://ngsir.netfirms.com/englishhtm/Lissajous.htm

http://www.math.com/students/wonders/lissajous/lissajous.html

Vocabulary:
Amplitude	Circle
Ellipse	Frequency
Lissajous figure	Pendulum motion
Plane	Sinusoidal equations
Square	Wave

Before:
- (*Level 1, 2, 3)* Pair students so that one person moves a pen left and right while the other moves the paper up and down. Compare the class' drawings.
- (*Level 1, 2)* Introduce the toy called Etch-A-Sketch. Try to draw a square and then try to draw a circle.

During:
- (*Level 1, 2, 3)* Each student should make one drawing and record the position of the weights and the cables. Adjust either the weights or the cables and make a second drawing. Record the difference in the drawings. Share what each student noticed to discover how the weights and cables affect the wave motion.

After:
- (*Level 3)* Study the following applet about Lissajous figures:
  http://ngsir.netfirms.com/englishhtm/Lissajous.htm
  How does changing the values of the frequency, initial phase, and amplitude affect the Lissajous figure?
  Students can view more complex Lissajous figures at
  http://www.math.com/students/wonders/lissajous/lissajous.html

Universal Wheel of Chance

Brief Description:
This exhibit is a large wheel with the numbers from 0 to 99 that visitors spin. The numbers are labeled with the same set of icons that are used on the Number Line. For example, the Fibonacci numbers are labeled with a rabbit and the primes are labeled with an atom. There is also a set of cards labeled with the same symbols (an atom, a rabbit, etc.).

Objectives:
Spinning the wheel is a great way to develop a sense of probability and explore the likelihood and distribution of outcomes. Students can calculate the probabilities for the outcomes they get, and they can also devise their own probability-based games to play.

Links to Websites:
http://mathmidway.org/Training/chance.php

http://www.shodor.org/interactivate/activities/RacingGameWithTwoDie/

Vocabulary:

Cake Numbers	Constructible Polygon Numbers
Counting Numbers	Cubes
Event	Factor
Factorials	Fibonacci Numbers
Highly Composite Numbers	Integers
Outcome	Pentagonal Numbers
Perfect Numbers	Pizza Numbers
Powers of Two	Primes
Probability	Squares
Tetrahedral Numbers	Triangular Numbers
Whole numbers	Zero

Before:
- (*Level 1, 2, 3)* The Universal Wheel of Chance is like a die with 100 sides, so you can do some activities with your students starting with six-sided dice. There's an interactive version of this activity here: http://www.shodor.org/interactivate/activities/RacingGameWithTwoDie/
- Students choose a possible sum of two six-sided dice (2-12) to be their number. When the dice are rolled, the students who have the total as their number get to move on the board. They continue to roll the dice until one player (or players) reaches the goal (See attached spreadsheet).
- (*Level 1, 2, 3)* Familiarize students with all of the different sets of numbers used in the Wheel of Chance: Primes, Squares, Triangular numbers, Perfect numbers, Powers of 2, Cubes, Highly Composite numbers, Pizza numbers, Cake numbers, Pentagonal numbers, Constructible Polygon numbers, and Tetrahedral numbers.

Dice Race!

Choose a number from 2-12. When the two dice are rolled, if the total is your number, place an X in the column under your number. Race against your classmates to see which number reaches the finish first.

2	3	4	5	6	7	8	9	10	11	12
					START					
					FINISH					

After you've finished the race, fill out this table. In each box place the sum of its row and column value to create an addition table. This shows all the possible outcomes when two dice are rolled. Each of the 36 outcomes is equally likely.

Dice	**1**	**2**	**3**	**4**	**5**	**6**
1						
2						
3						
4						
5						
6						

Questions to discuss: Do you see a pattern here? What does this pattern have to do with the winner of the dice race?

The diagonal pattern shows that 7 should occur most often and 2 and 12 least often, with the others in between. This means the students who choose 7 are the likeliest to win the race, although it's possible for someone else to win because unlikely things can happen!

During:

- (*Levels 1, 2, 3*) Ask students to study the Wheel of Chance writing down possible outcomes that we can study, such as odd numbers and even numbers, symbols, number without a symbol vs. number with a symbol, etc. Predict the distribution of the outcomes and then make a list of outcomes. As students spin the Wheel of Chance, record the number of times it lands on the possible outcomes.

After:

- (*Levels 1, 2, 3*) Use the data that the students gathered to compare predicted distribution of outcomes with actual distribution of outcomes.

Pirate X and Lady Y

Brief Description:

The pirate ship acts as a balance. On each side, there are three posts, spaced at equal intervals, from which the participant can hang weights. A weight on post 2 has twice the effect of one on post 1, and a weight on post 3 has 3 times the effect. Participants can choose from brass disks marked with weights ranging from 1 to 10. The goal is for the two sides of the ship to balance.

Objectives:

Algebraic equations are fun to solve at this tactile exhibit. Students set up equations and demonstrate solutions by literally keeping everything balanced. They can also translate their set-ups into algebraic notation and discover the many strategies for solving equations. An equation or inequality can be modeled here. Students can even learn about solving equations with two variables by using both Pirate X and Lady Y in their equations. Additionally, students discover that for a fulcrum to maintain balance the distribution of weight multiplied by the distance to the fulcrum must be the same on each side.

Links to Websites:

http://mathmidway.org/Training/pirate.php

http://nlvm.usu.edu/en/nav/frames_asid_201_g_4_t_2.html?open=instructions

http://www2.stetson.edu/~efriedma/weight/

Vocabulary:

Algebraic notation	Equation
Fulcrum	Inequality
Torque	Variable

Before:

- (*Levels 2, 3*) The following website is an interactive introduction to the solving of equations. http://nlvm.usu.edu/en/nav/frames_asid_201_g_4_t_2.html?open=instructions
- (*Levels 1, 2, 3*) Constructing coin mobiles: students will use a wire hanger to place different weights of coins so it will balance evenly.

1. Collect the following materials: wire hangers, coins, plastic sandwich bags, and string.
2. Demonstrate by hanging the wire hanger from a piece of string.
3. Place coins in a sandwich bag and place at one end of the wire hanger. Notice how the wire hanger will dip to one side.
4. Have students discuss what should be placed on the other side to get it balanced. See what works.
5. Give students a chance to make their own balanced coin mobile. One may replace coins with standardized weights.

During:

- (*Levels 1, 2, 3*) Take a single weight and place it on the 1 hook. Notice how the ship "sinks." Ask students to think about what would happen if they placed the single weight on the 2 hook.
  Would the ship sink more, less, or the same as being on the 1 hook?

Set up random weights that are unbalanced. Ask students to figure out the weight to balance the ship. Write the representation as an equation.

- (*Levels 2, 3*) Set up two different equalities with one ***x*** and one ***y***. Make sure to use the same ***x*** and ***y*** for both. Ask students to figure out what the ***x*** and ***y*** must weigh.
- (*Levels 1, 2, 3*) The training video associated with this exhibit includes many suggestions for student activities.

After:

- (*Levels 1, 2, 3*) Nested balance puzzles, including a discussion of torque is found at: http://www2.stetson.edu/~efriedma/weight/
  Ask students to record the equation that balances the weights.

Magician and the Moon

Brief Description:
The Magician and the Moon exhibit deals with covering a circle with five smaller circles.

Objectives:
This covering problem is a fun challenge involving the area of circles. Students can discuss their strategies for solving this problem, and then use hands-on exploration to discover the counter-intuitive solution.

Links to Websites:
http://mathmidway.org/Training/moon.php

http://demonstrations.wolfram.com/TheCircleCoveringPuzzle/

Vocabulary:

Area of a circle	Circumference
Diameter	Pi
Radial symmetry	Radius

Before:
- (*Levels 1, 2*) It's an interesting activity to cover a circle with six smaller circles. It can be done on paper with a compass.

- (*Levels 2, 3*) Given three congruent circles, arrange them in such a way as to cover the largest possible fourth circle.

During:
- (*Levels 1, 2, 3*) Have your students solve the problem of covering the white circle with 5 smaller circles. For a hint, you can first tell them that the answer is not completely symmetric. As a second hint, note that the outer points where overlapping magnets touch each other all sit on the circumference of the big circle.

After:
- (*Levels 1, 2, 3*) Examine other covering problems online. Download the free Mathematica Player and try other covering problems here: http://demonstrations.wolfram.com/TheCircleCoveringPuzzle/

- (*Levels 2, 3*) *Investigation:* Consider the relationship between the measure of the radius of the big circle and that of the small circle.

Three Ring Circleous

Brief Description:

Three objects are spinning so quickly that you only see a blur. The three objects are: a line that rotates to make a hyperboloid, a cube that rotates to make a combination of cones and hyperboloid, and three ellipses that rotate to make a cylinder. Pushing a button stops the spinning to reveal the object.

Objectives:

This exhibit encourages students to reason spatially as they observe the three-dimensional shape swept out by a spinning object. Students can predict what shapes are spinning to create the visual image of the solid, or they can figure out what solids those shapes might make if rotated differently.

Links to Websites:

http://mathmidway.org/Training/circleous.php

http://www.fi.uu.nl/toepassingen/00182/toepassing_wisweb.en.html

http://scratch.mit.edu/projects/AddZero/152472

http://www.georgehart.com/skewers/skewer-hyperboloid.html

Vocabulary:

Cone	Cube
Cylinder	Hyperboloid
Lathe	Solid of revolution

Before:

- (*Level 3*) Play with solid of revolution apps online. Both of these websites allow you to manipulate a two-dimensional shape and see the results of rotating it into a solid.
  http://www.fi.uu.nl/toepassingen/00182/toepassing_wisweb.en.html
  http://scratch.mit.edu/projects/AddZero/152472

During:

- (*Level 2, 3*) Here are three questions to ask your students at this exhibit.
  Can you predict what shape is spinning before anyone hits the button to stop the rotation?
  Can you think of other shapes that would make the same solid if spun? (Example answers: a vertical rod spinning at the edge of the disc would also make a cylinder. A spring shape (helix) would make a cylinder. A straight line could make a cone. A curved rod could make the hyperbolic vase.
  If you took these same shapes and rotated them a different way, what kinds of solids might you see? (Example answer: With the rod in different orientations, it could also be a cylinder or two cones touching at their points.)

After:

- (*Level 3*) Make a hyperbolic sculpture using bamboo skewers.
  http://www.georgehart.com/skewers/skewer-hyperboloid.html