Organ Function Grinder

Brief Description:

At this exhibit, students can explore the concept of functions in a fun musical context. Students can investigate many aspects of this topic, including estimating outcomes, figuring out which input and functions will yield a desired outcome, composing functions, inverting functions, and graphing or charting the outcome of several inputs under the same function.

Objectives:

Students will understand that a function is a relationship between values where each input value determines exactly one output value. Students will conclude that there are multiple ways of arriving at a value. They will examine inverse relationships between values and discover the associative and commutative properties of addition and multiplication. For more advanced levels, students will connect musical composition with transformation geometry.

Links to Websites:

http://mathmidway.org/Training/organ.php

http://www.classicsforkids.com/terms/

http://score.kings.k12.ca.us/lessons/functions.htm

http://www.regentsprep.org/Regents/math/algtrig/ATP5/Lfunction.htm

Vocabulary:

Associative Property	Commutative Property
Distributive Property	Double
Function	Halve
Identity Element	Input
Inverse operation	Invert
Order of Operations	Output
Reciprocal	Square
Square Root	Undefined

Before:

- (*Level 1, 2, 3*) Teacher should explain the 8 available functions (add 3, subtract 3, square, square root, double, halve, invert, leave alone) or any of the functions appropriate to the group.

  After talking about function composition, practice composing functions. Group students into teams and give each team an output number. If your students are not familiar with function notation, you can make function machine diagrams (i.e. operations on the organ grinder) out of paper and give one to each student. Students will hold their diagrams arrange themselves to reach goal outputs using an input of their choice. First team to reach all goal outputs wins.

- (*Levels 1, 2*) Sometimes the order of the function matters! Discover when it does with the following examples:

Using the functions "*add 2*," "*multiply by 3*," and "*subtract 7*," determine an order in which they should be performed to get the required output from the given input.
input: 2 output:1 (ans. *multiply by 3, add 2, subtract 7* or *multiply by 3, subtract 7, add 2)*)
input: 7 output: 6 (ans. *subtract 7, add 2, multiply by 3* or *add 2, subtract 7, multiply by 3)*)
input: 5 output: 10 (ans. *multiply by 3, subtract 7, add 2* or *multiply by 3, add 2, subtract 7*)

Can you identify the operations when order doesn't matter?
Are these the only solutions?

Using the functions "*add 2*," "*multiply by 2*," and "*square*," determine the order they should be performed to get the output from the input.

input: 1 output:36 (ans. *add 2, multiply by 2, square,*)
input: 1 output: 16 (ans. *multiply by 2, add 2, square*)
input: 1 output: 18 (ans. *add 2, square, multiply by 2*)
input: 1 output: 4 (ans. *square, multiply by 2,add 2)*

During:

- (*Level 1, 2, 3*) Each student in a group of four is assigned one of the Function Grinder's operations. As a relay race a number is given to the first person in the group who performs the operation and writes the answer down to pass to the next person. The final result is given to a student to check by using the Organ Function Grinder. Students can rearrange themselves to get different results.

  Select 8 students to represent each of the functions on the Organ Function Grinder and have them stand. Write the function on a card that each student will hold. Docent selects an input number ticket and sets the knobs without letting the students see. Docent shares the result. Students must work to arrange three of the standing students in inverse order to produce the input number on the ticket. [Some students may catch on that each function has a distinctive sound.]

  Use the attached sheets of goal values. Give a sheet to each group of students, and they can try to find the input values and functions for as many of the outputs as they can. Students can select a few of their functions to test on the Organ Function Grinder.

- (*Level 3*) Investigations: Bobson Wong, Mathematics Teacher at Bayside High School in New York, is a Master Teacher with Math For America. He has submitted the attached plans and worksheets to the Museum of Mathematics to relate mathematical transformations to musical transformations. The plan is referenced on momath.org, from a lesson plan written for Math For America, titled "Field Trip on Functions."

- (*Level 2*) Use the lesson plan, designed by the Math Midway, for grades 4-9, accessible at momath.org.

After:

- (*Level 1, 2, 3*) Using the worksheet "Organ Function Grinder Activity: Reach the Goal!" study the ordering of functions and its effect on output values.

  Separate the class into groups or have students work individually.
  Students select 10 input values and then three functions.
  Students rearrange functions to examine the number of possible outputs.
  Pose the question: *"Whose inputs and function choices get the most outputs?*".

- (*Level 2, 3*) Create a worksheet that contains the following:
  What is a function?
  Why are the eight operations defined as functions?
  Create your own composition of functions using given inputs and determine if the composition is commutative.

Organ Function Grinder Activity: Reach the Goal!

Part I: Find the input and settings for the knobs that will give you the goal output. For the first chart, 2 of the knobs must be set to "leave alone."

Goal Output	Input	Knob 1	Knob 2	Knob 3
100				
20				
7.5				
49				
8				
0.5				
19				
78				
30				
1/9				

Part II: Using two functions and one dial set to "leave alone:"

Goal Output	Input	Knob 1	Knob 2	Knob 3
1/20				
11				
222				
17				
¼				
400				
-2.5				
-2				
84				
25				

Part III: Using three functions (no dials set to "leave alone"):

Goal Output	Input	Knob 1	Knob 2	Knob 3
403				
3 ¼				
124				
4/49				
17.5				
-10				
¼				
27				
3 1/9				
9997				

Answers (note there may be other possible answers)

Part I: Find the input and settings for the knobs that will give you the goal output. For the first chart, 2 of the knobs must be set to "leave alone."

Goal Output	Input	Knob 1	Knob 2	Knob 3
100	10	Square	leave alone	leave alone
20	10	Double	leave alone	leave alone
7.5	15	Halve	leave alone	leave alone
49	7	Square	leave alone	leave alone
8	64	square root	leave alone	leave alone
0.5	1	Halve	leave alone	leave alone
19	16	add three	leave alone	leave alone
78	81	subtract three	leave alone	leave alone
30	15	Double	leave alone	leave alone
1/9	9	Invert	leave alone	leave alone

Part II: Using two functions and one dial set to "leave alone:"

Goal Output	Input	Knob 1	Knob 2	Knob 3
1/20	10	Double	invert	leave alone
11	4	Double	add three	leave alone
222	15	Square	subtract three	leave alone
17	10	Double	subtract three	leave alone
¼	2	Invert	square	leave alone
400	10	Double	square	leave alone
-2.5	-8	add three	halve	leave alone
-2	2	Halve	subtract three	leave alone
84	9	Square	add three	leave alone
25	2	add three	square	leave alone

Part III: Using three functions (no dials set to "leave alone"):

Goal Output	Input	Knob 1	Knob 2	Knob 3
403	10	Double	square	add three
3 ¼	2	Invert	square	add three
124	8	add three	square	add three
4/49	7	Halve	invert	square
17.5	16	Double	add three	halve
-10	1	subtract three	subtract three	double
¼	4	Halve	invert	square
27	9	add three	double	add three
3 1/9	3	Invert	square	add three
9997	10	Square	square	subtract three